Measuring Nestedness

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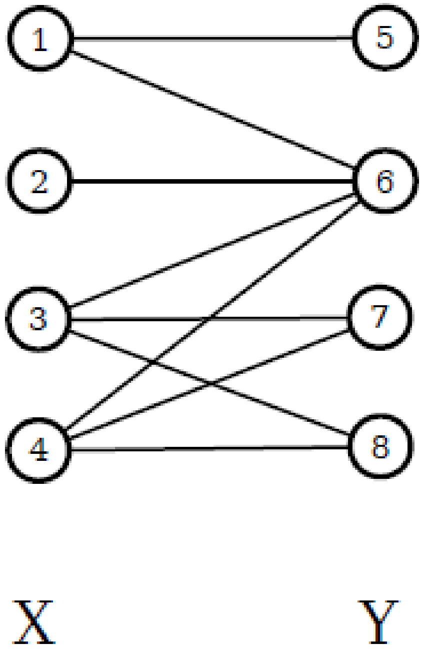
Bipartite networks



Two sets (groups) of nodes (X and Y).
There are only connections between nodes that do not belong to the same set.

Adjacency matrix:

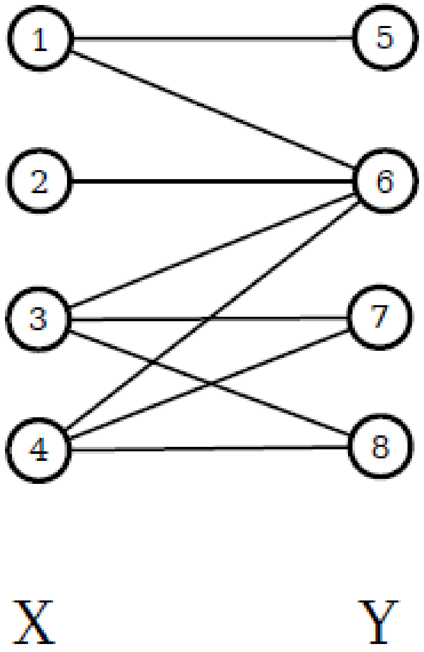
$$A = \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

Incidence matrix:

$$B = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

$$B_{i,j} = \begin{cases} 1 & \text{if node i and node j are connected} \\ 0 & \text{otherwise} \end{cases}$$

Bipartite networks



Two sets (groups) of nodes (X and Y).
There are only connections between nodes that do not belong to the same set.

n_x - number of nodes in set X (rows)
n_y - number of nodes in set Y (columns)

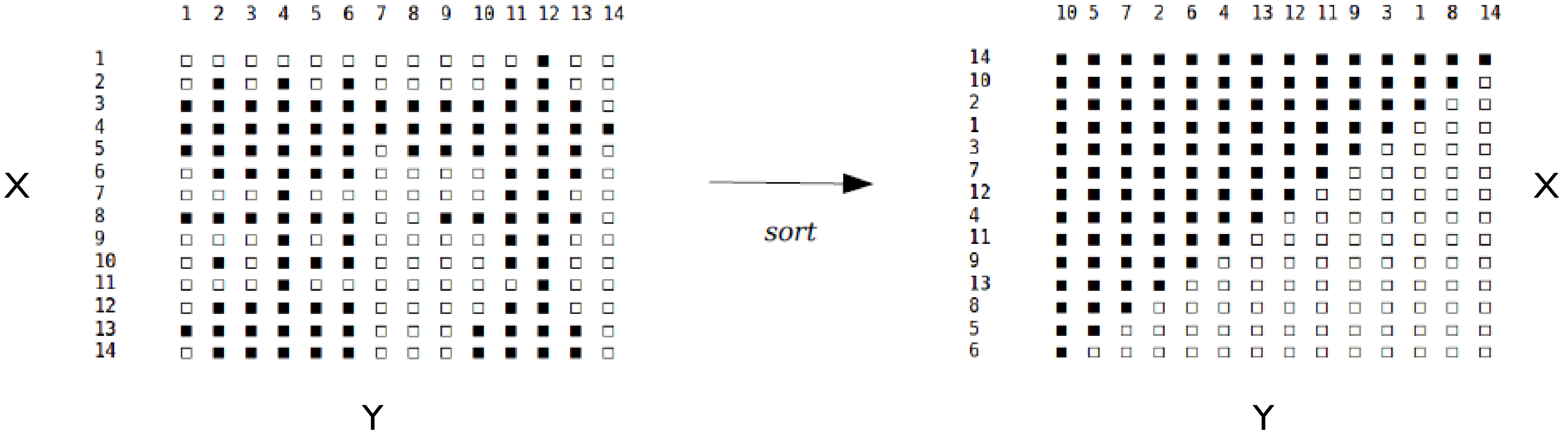
m - number of edges in the graph

Connectance C of a bipartite networks is given by:

$$C = \frac{m}{n_x n_y}$$

Nestedness

Sort columns and rows of the incidence matrix by the degrees of the nodes:



A network is nested, if for both groups X and Y:

1) there are nodes with many interactions (generalists) and nodes with a few interactions (specialists)
2) the nodes with few interactions share the interactions with the nodes with many interactions

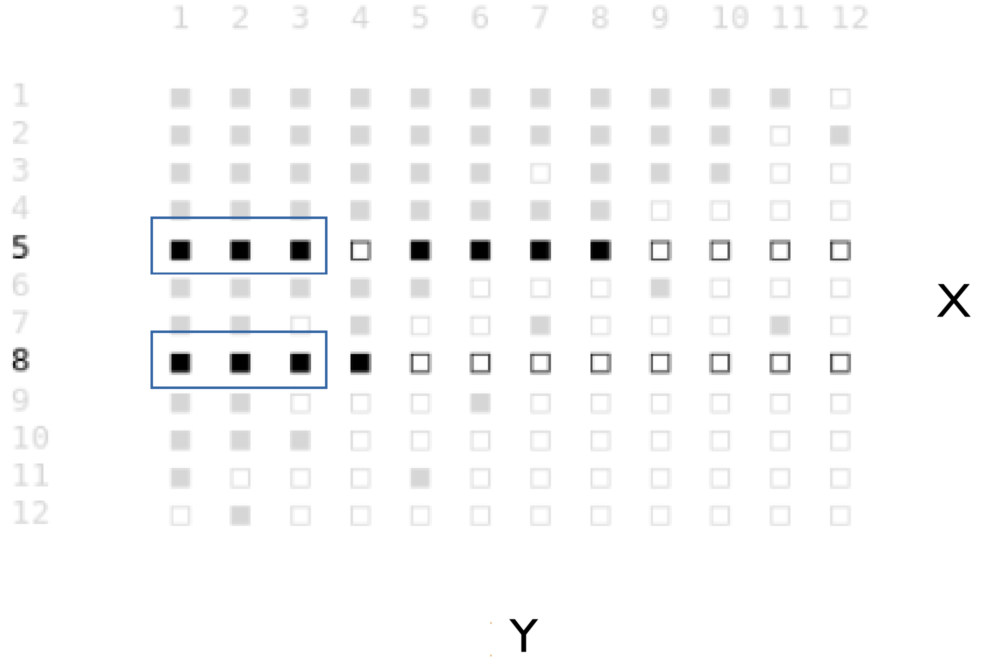
Calculating nestedness – Fortuna et al. (2019)

The overlap o_{ij} between two nodes i and j (from the same group) is the fraction of interactions of the node with the smaller degree that are shared by the node with the larger degree.

$$o_{ij} = \frac{c_{ij}}{\min(k_i, k_j)}$$

c_{ij} - the number of interactions node i and j share

$$c_{ij} = \sum_{k=1}^{n_y} B_{i,k} B_{j,k} \qquad \text{(for rows)}$$



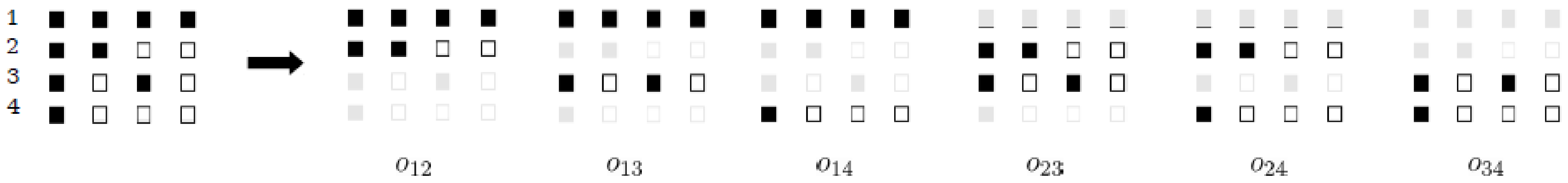
Example: overlap between node 5 and node 8 is:

$$o_{5,8} = \frac{3}{\min(7{,}4)} = \frac{3}{4}$$

Calculating nestedness – Fortuna et al. (2019)

Step 1: calculate the overlap of all pairs of rows

$$\sum_{i=1, i<j}^{n_x} o_{ij}$$



Step 2: calculate the overlap of all pairs of columns

$$\sum_{i=1, i<j}^{n_y} o_{ij}$$

Calculating nestedness – Fortuna et al. (2019)

Step 3: calculate nestedness N of the network – the average overlap of all pairs of rows and all pairs of columns:

$$N = \frac{\sum_{i=1,i<j}^{n_x} o_{ij} + \sum_{i=1,i<j}^{n_y} o_{ij}}{\frac{n_x(n_x - 1)}{2} + \frac{n_y(n_y - 1)}{2}}$$

N has values between 0 (not nested) and 1 (perfectly nested).