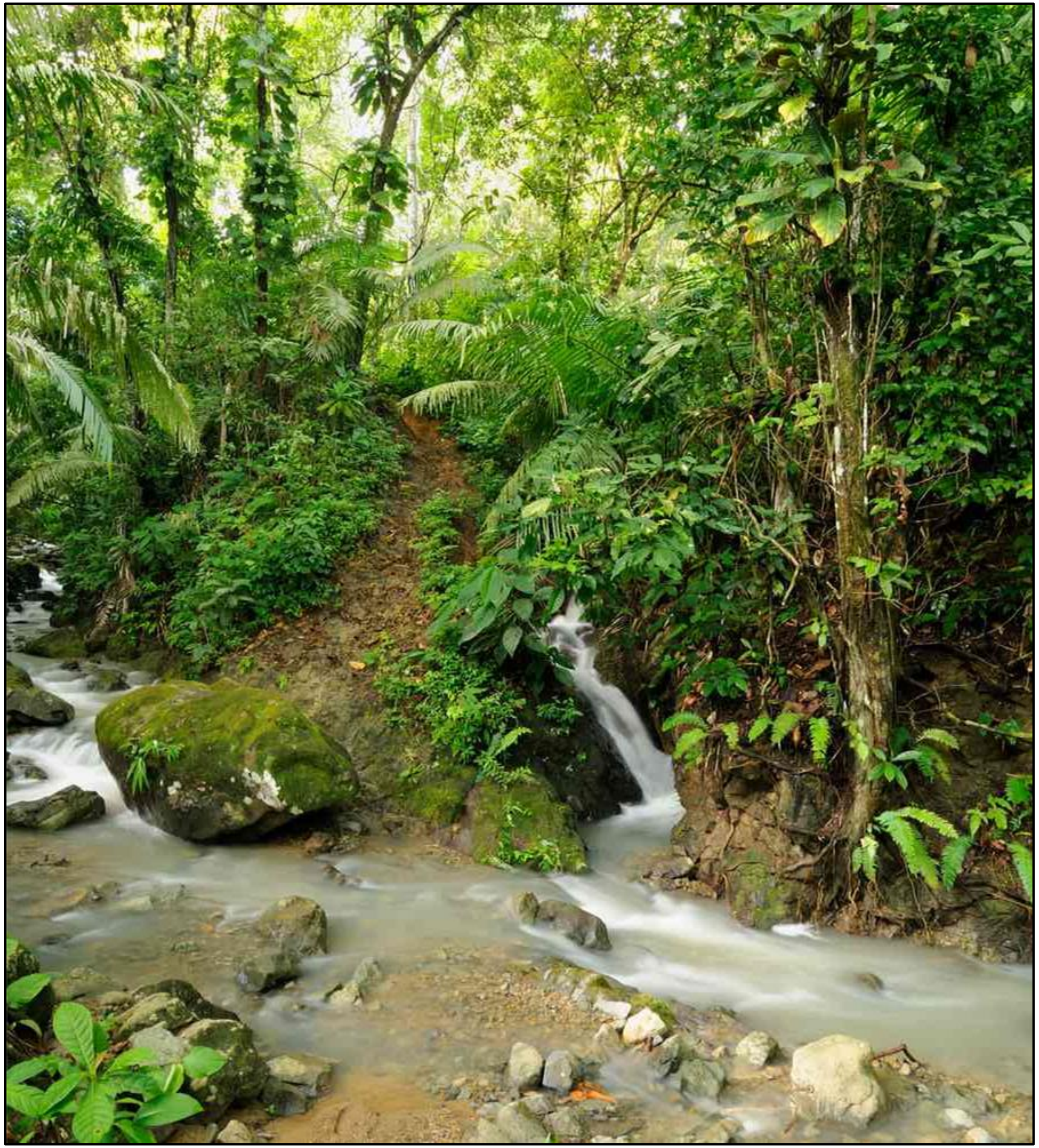
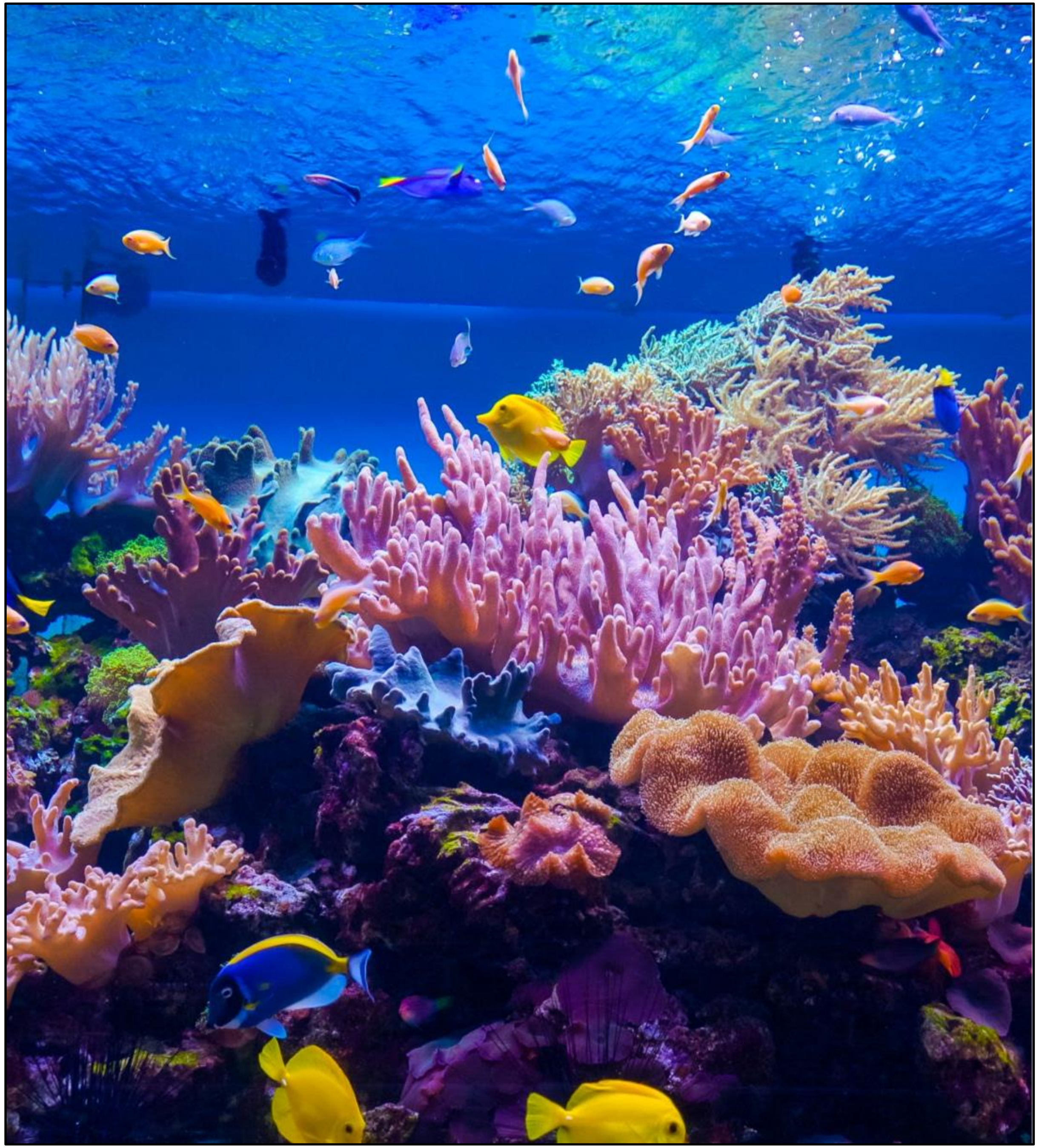
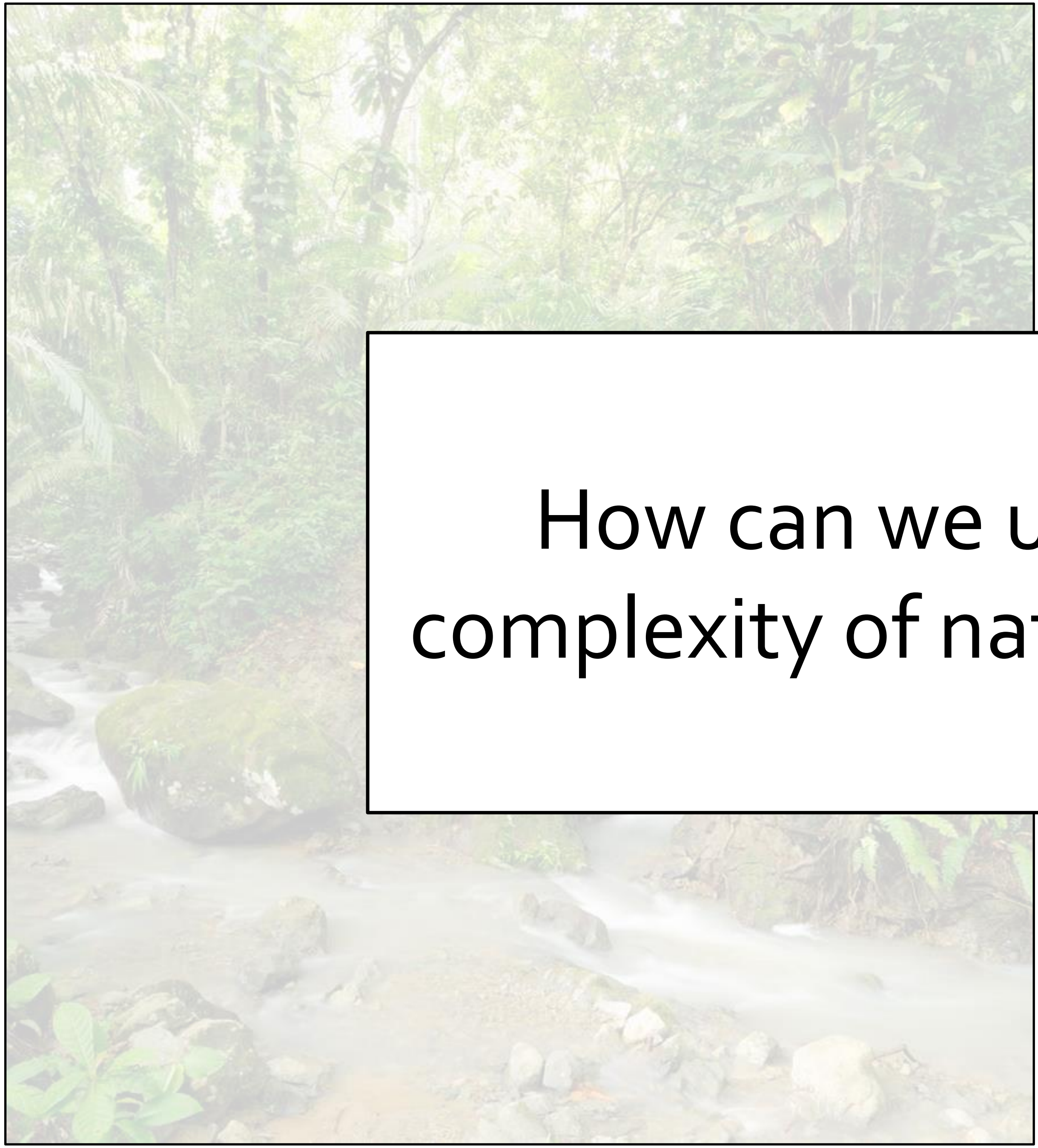
The role of species in networks

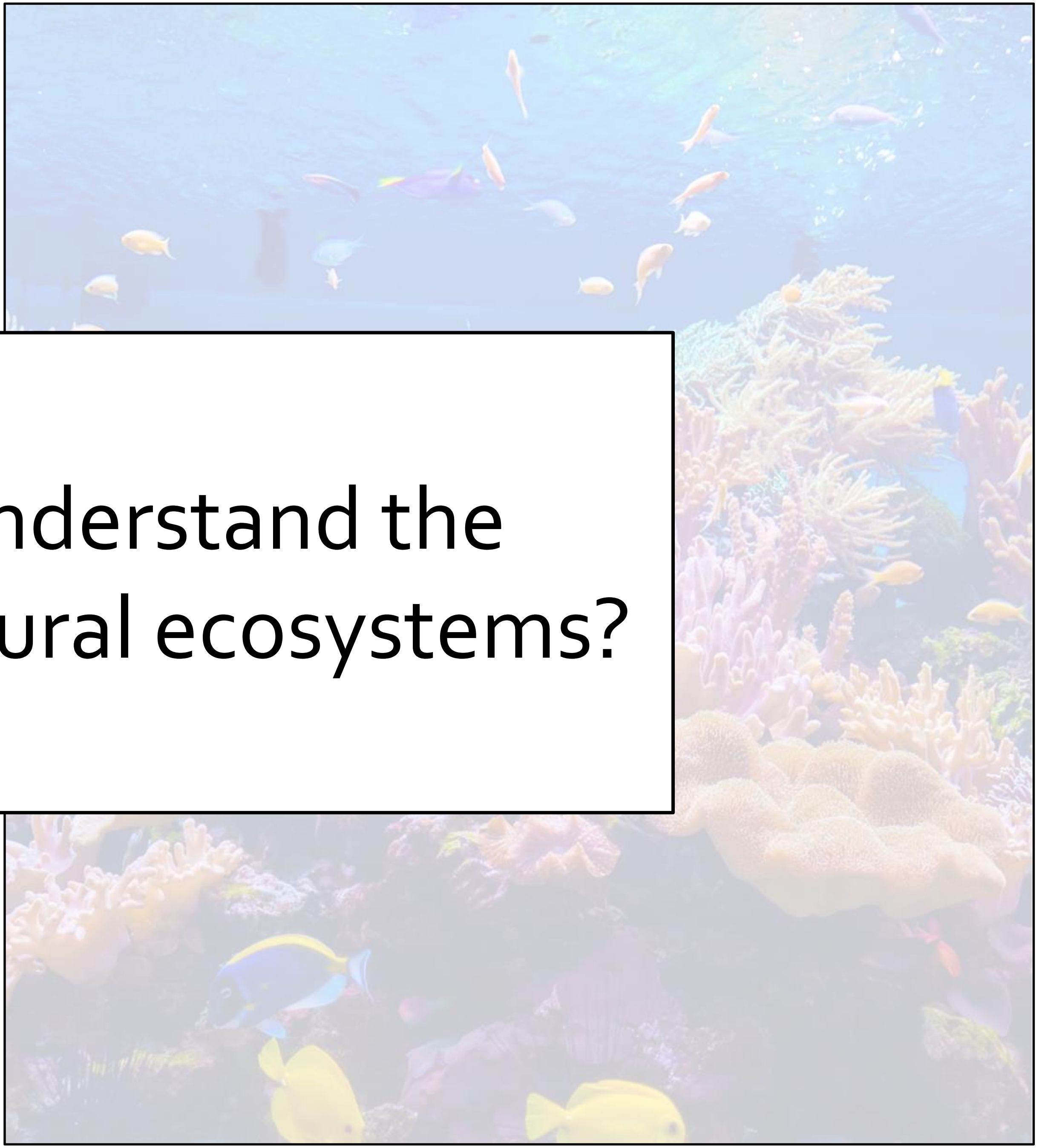
BIO365 – Ecological Networks

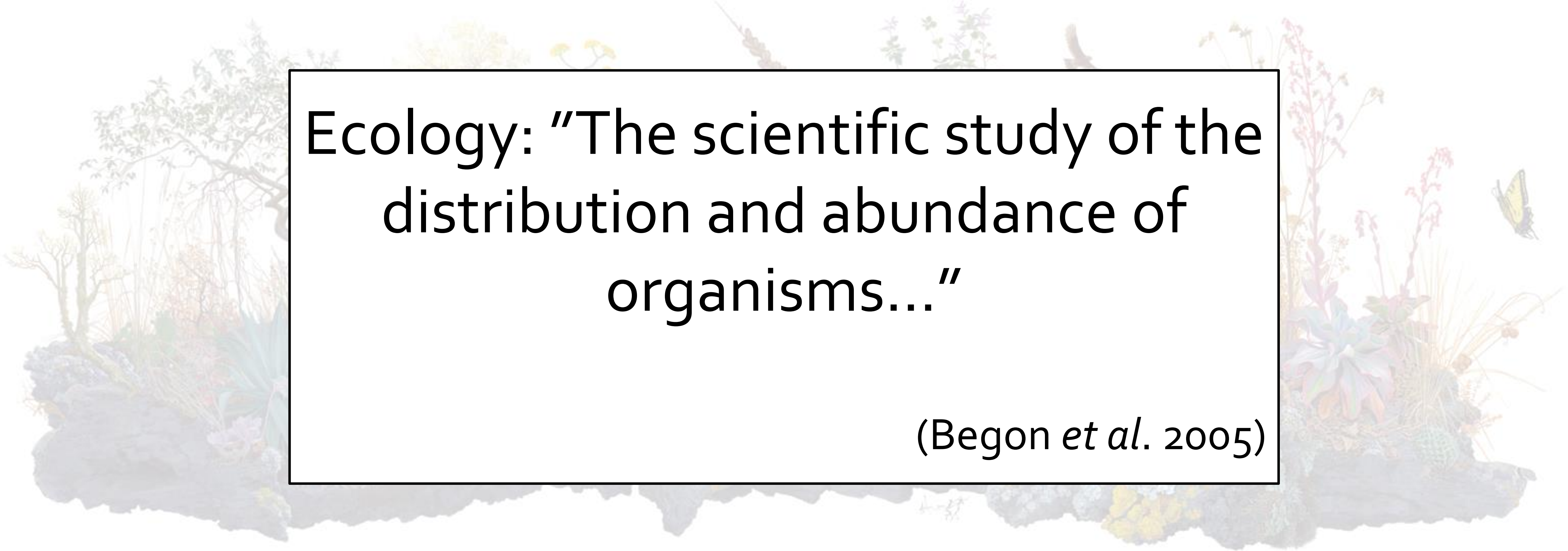
Leandro G. Cosmo
leandro.giacobellicosmo@uzh.ch



How can we understand the complexity of natural ecosystems?





Ecology: "The scientific study of the distribution and abundance of organisms…"

(Begon *et al*. 2005)

Richness and abundance of species: species diversity

Is quantifying species diversity is enough?

Species are not independent from each other!

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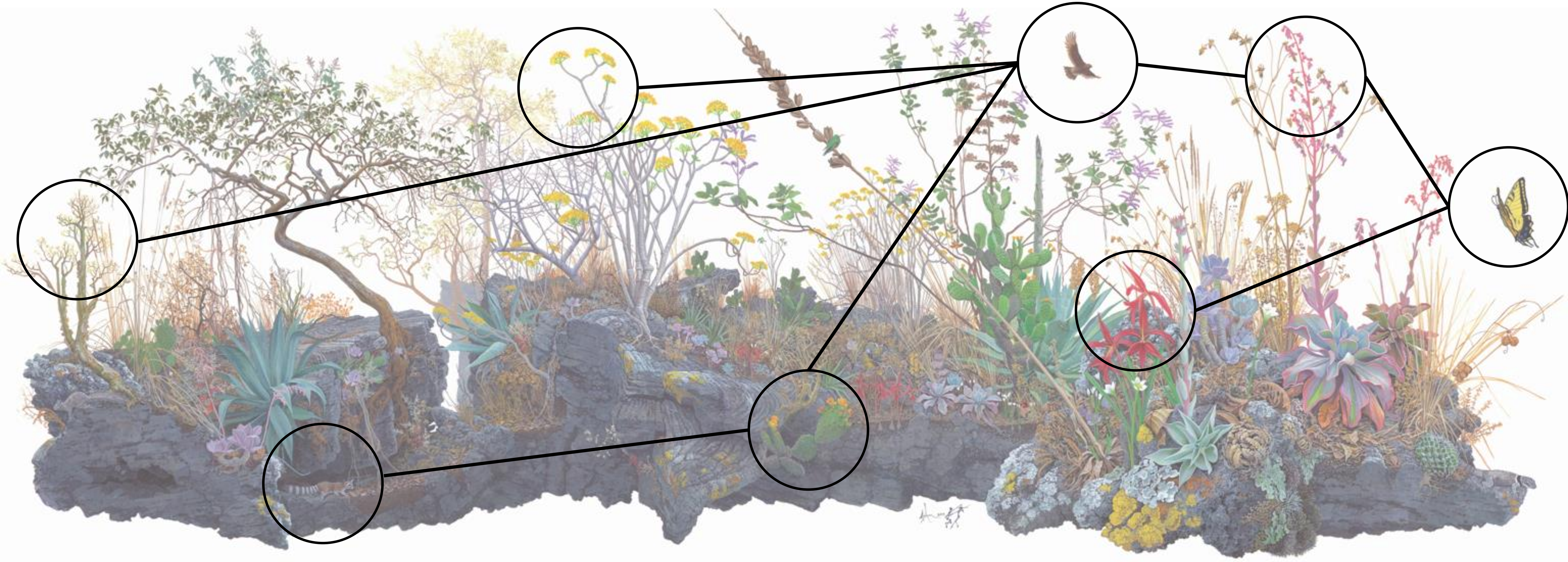

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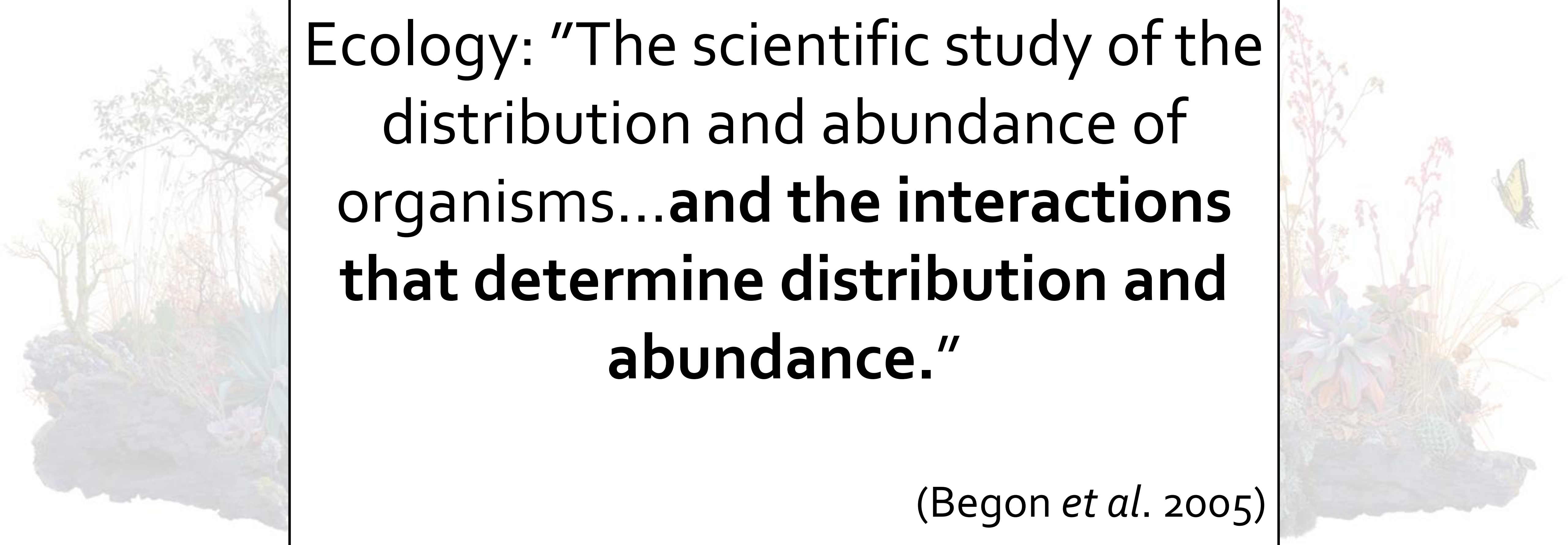
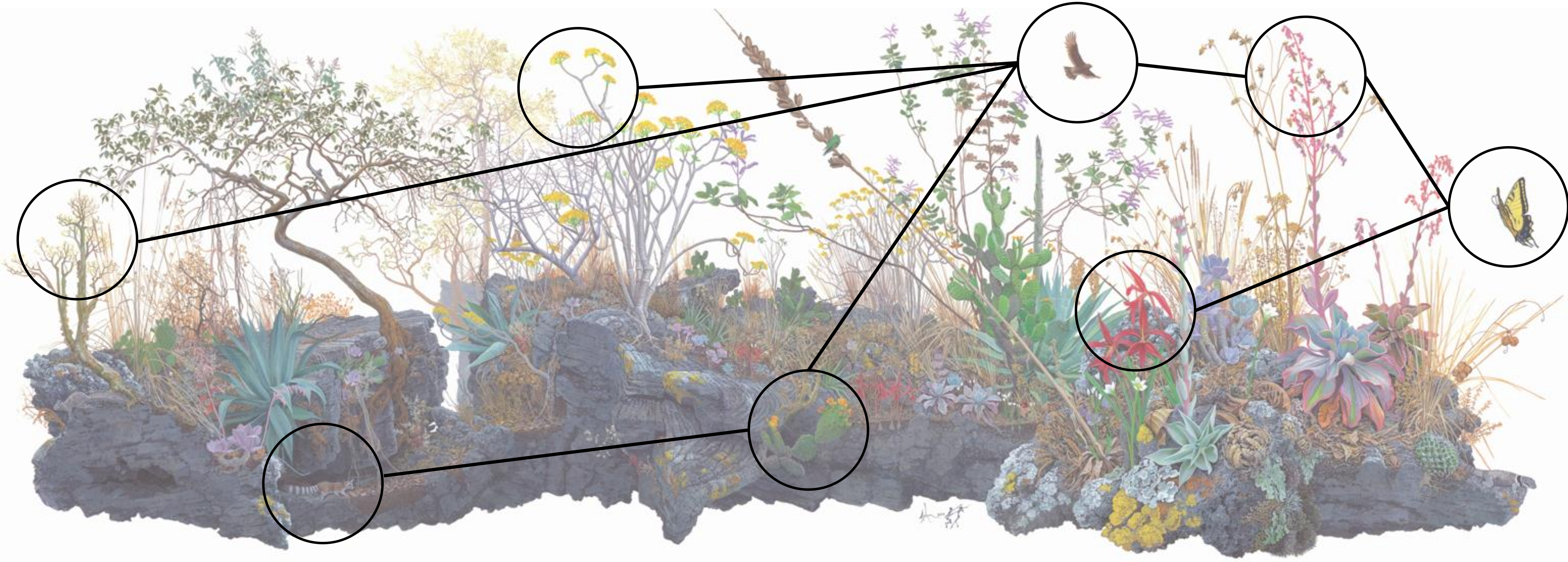
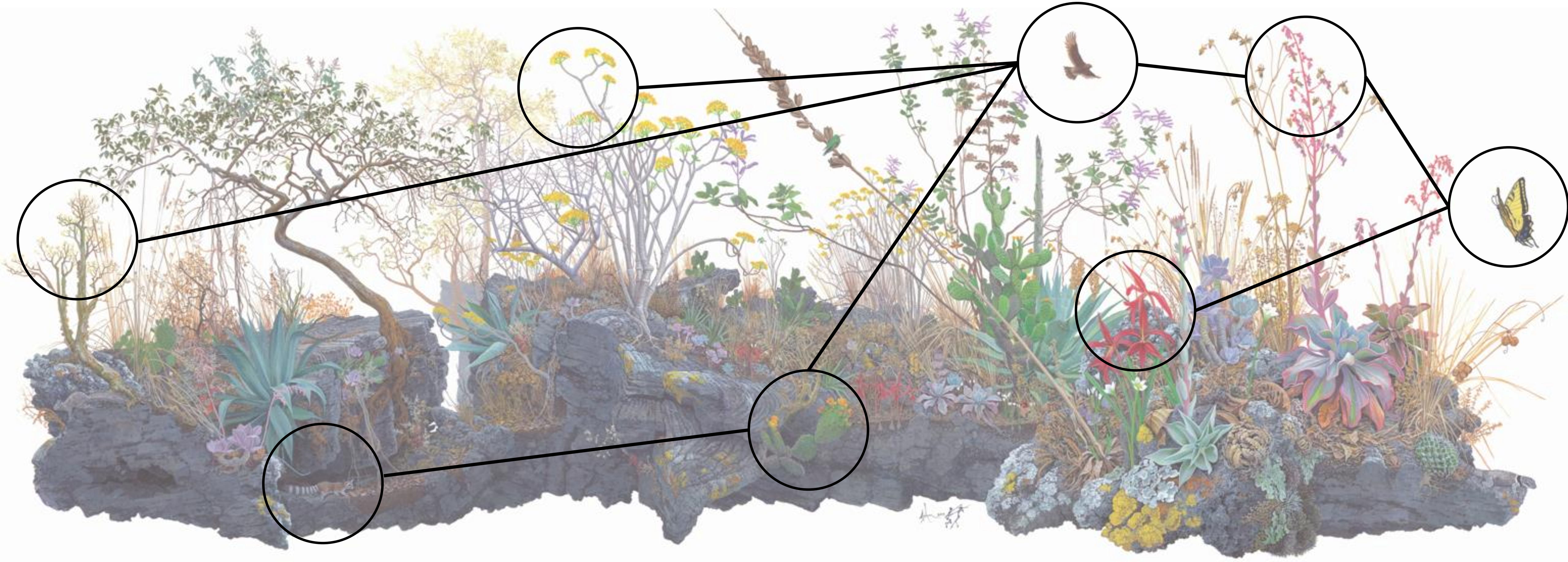


Ecology: "The scientific study of the distribution and abundance of organisms…**and the interactions that determine distribution and abundance.**"

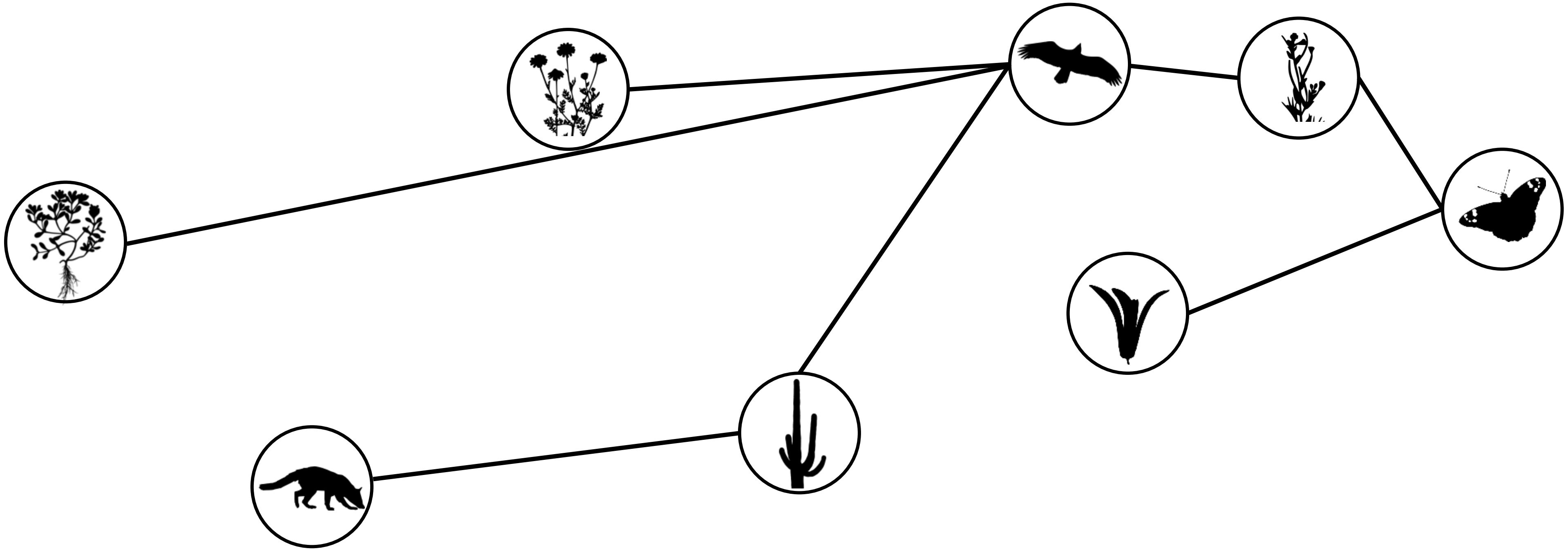
(Begon *et al*. 2005)

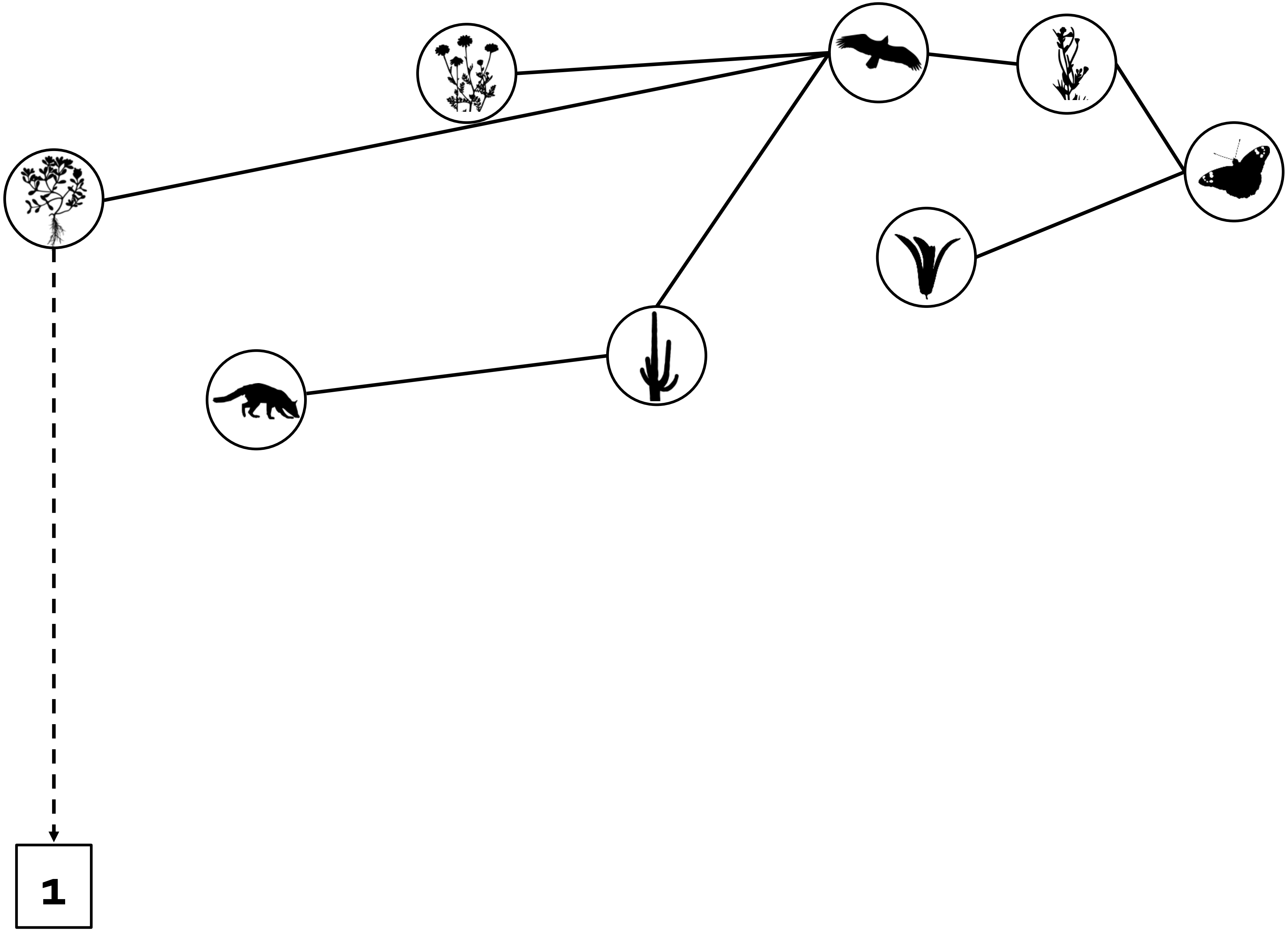


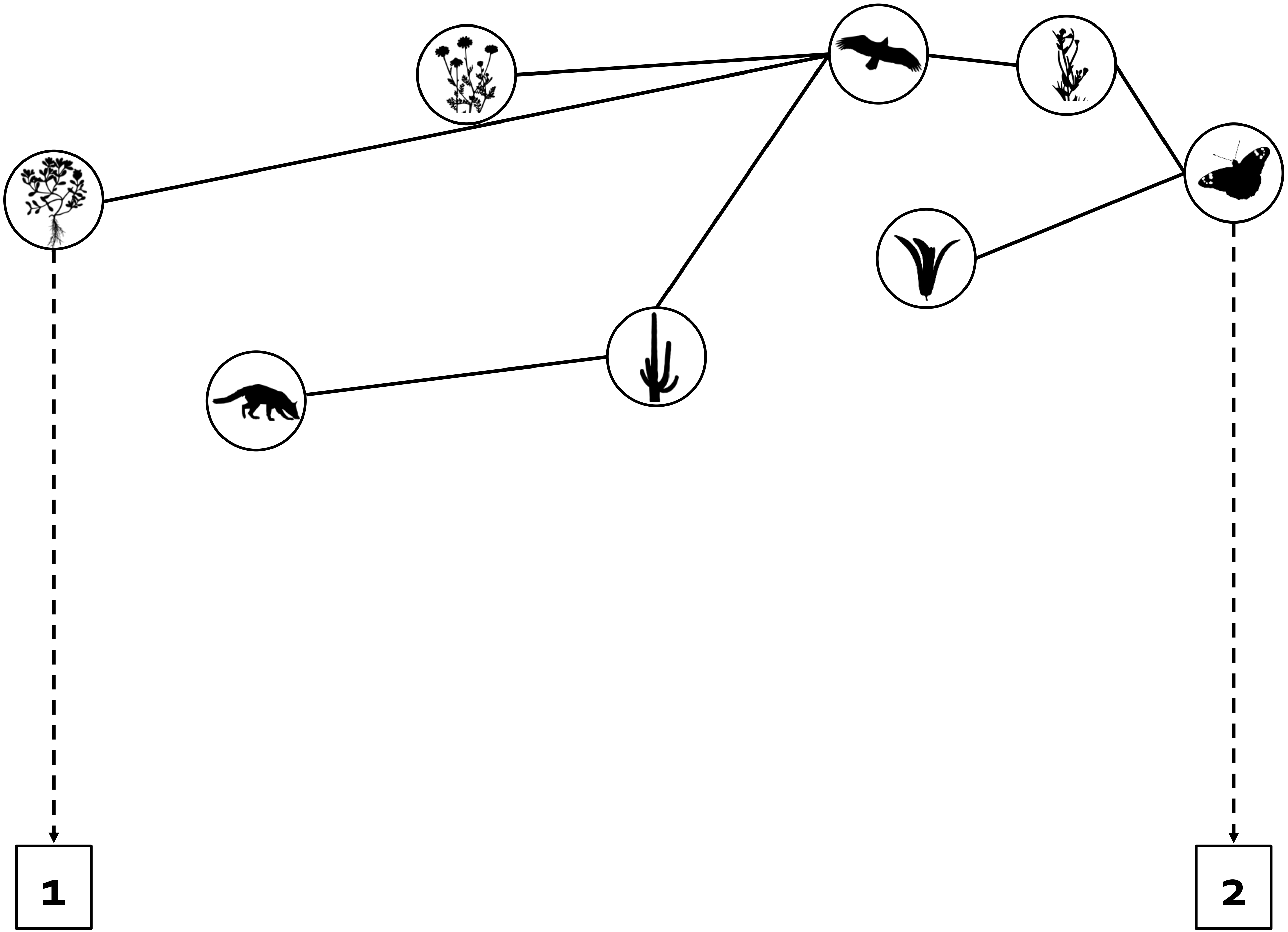
How can we quantify patterns of interactions?

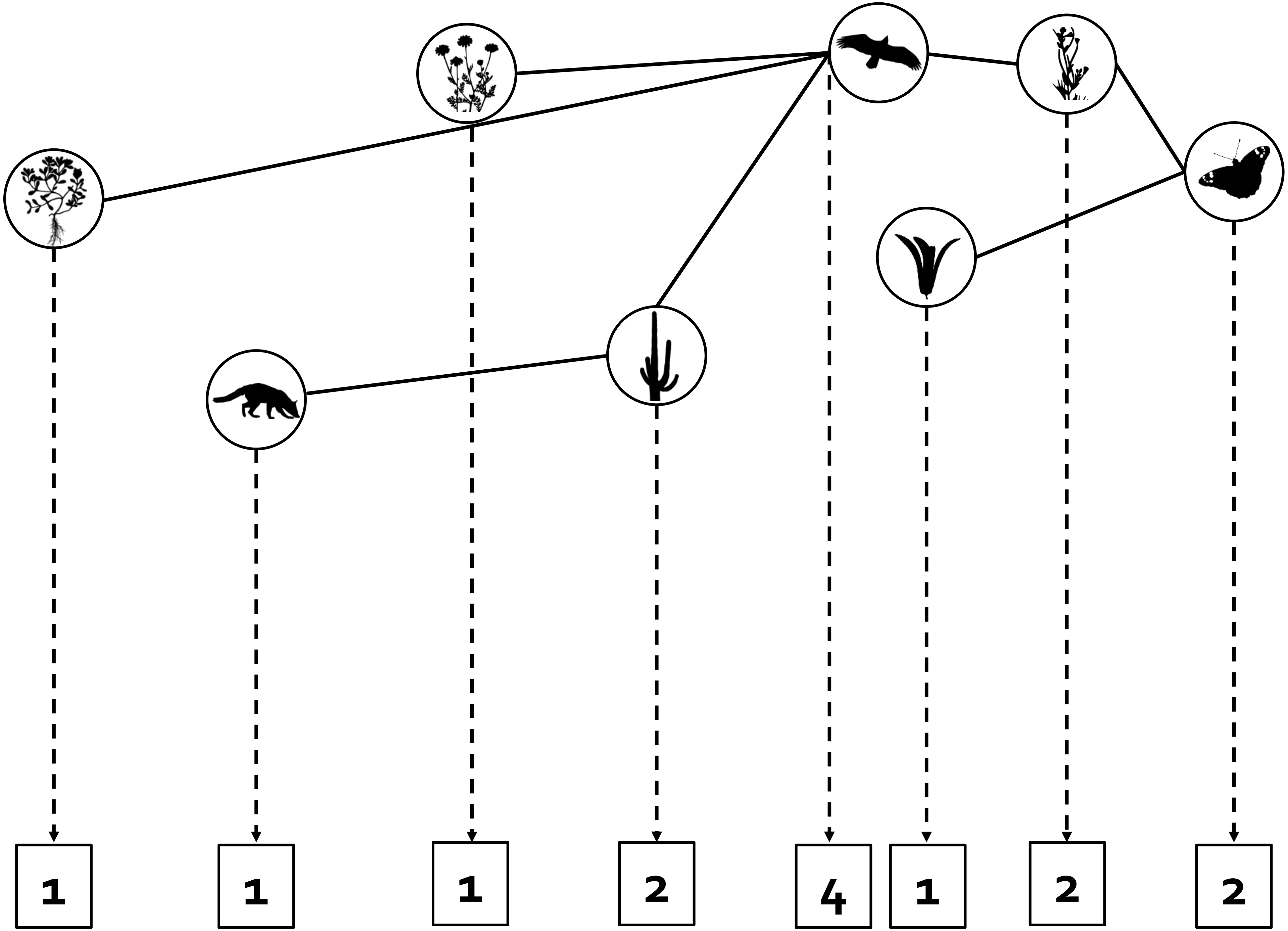


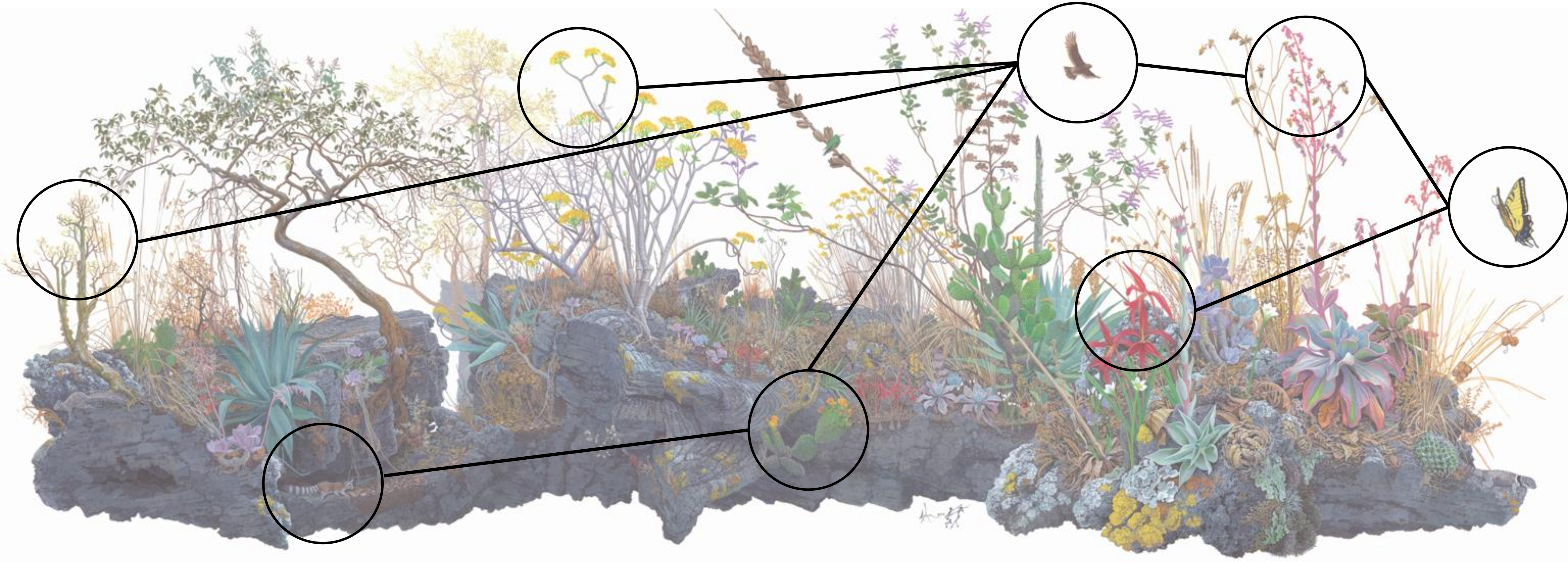
How many interactions each species establish?



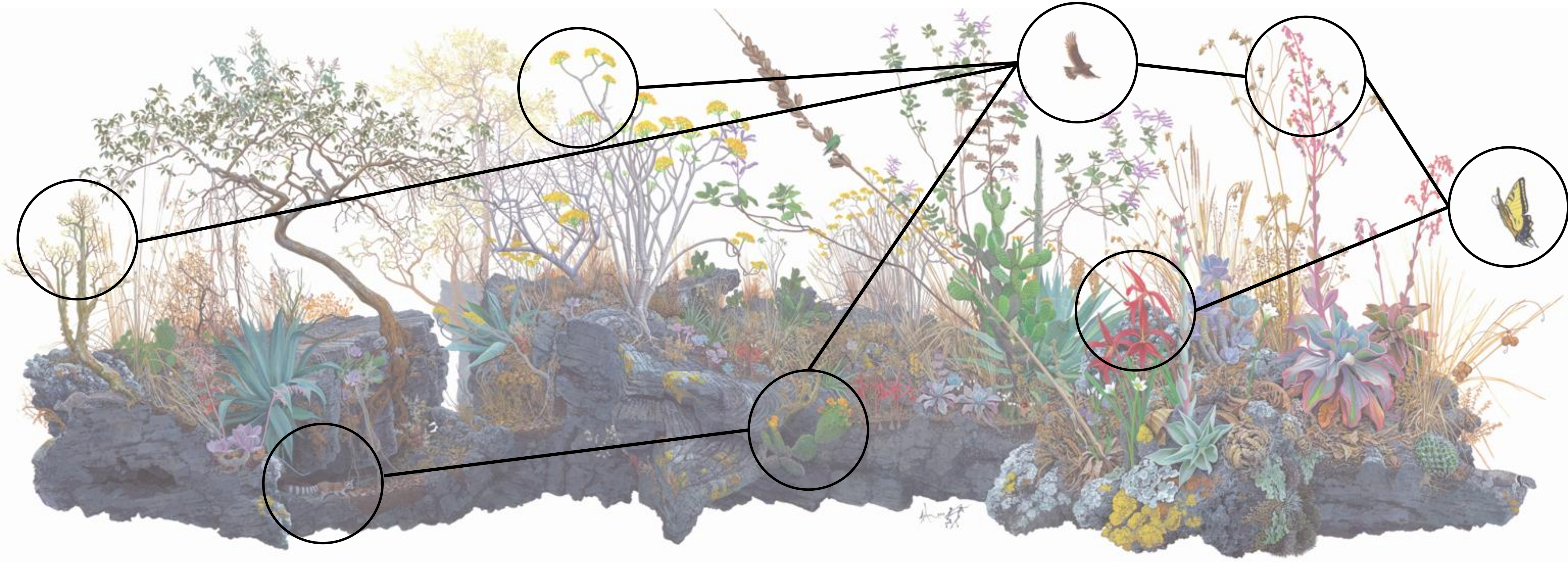

1


1
2


1
1
1
2
4
1
2
2



Number of interactions of each species: *species degree*



We can quantify degree from the interaction matrix

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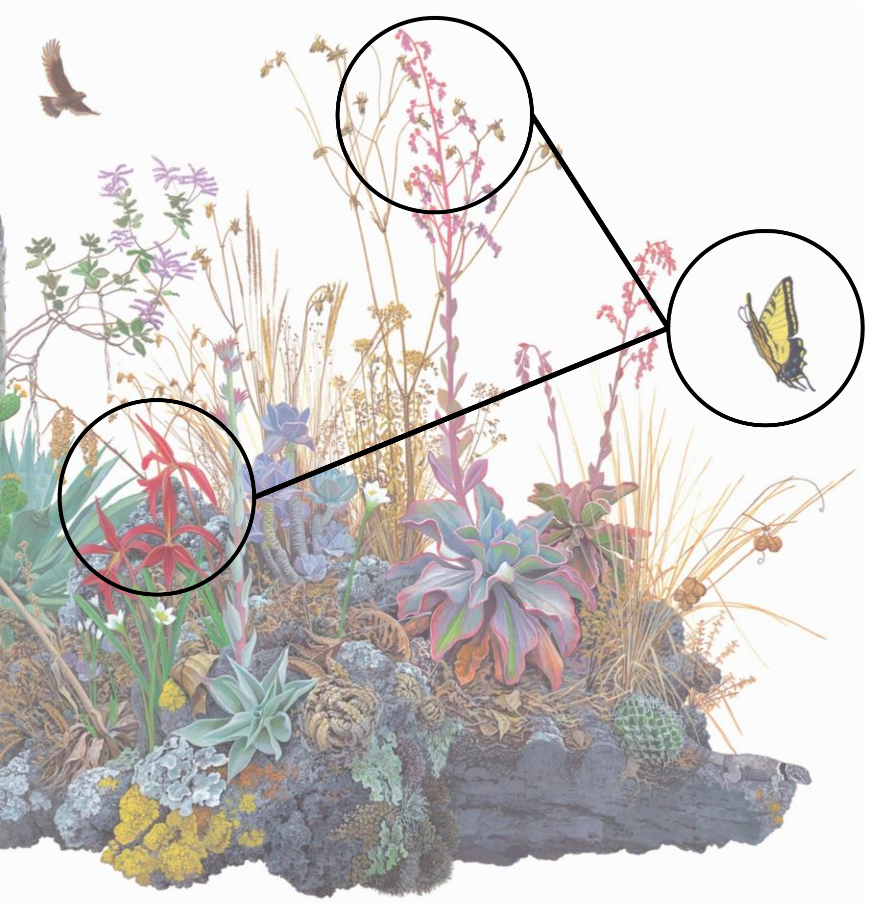
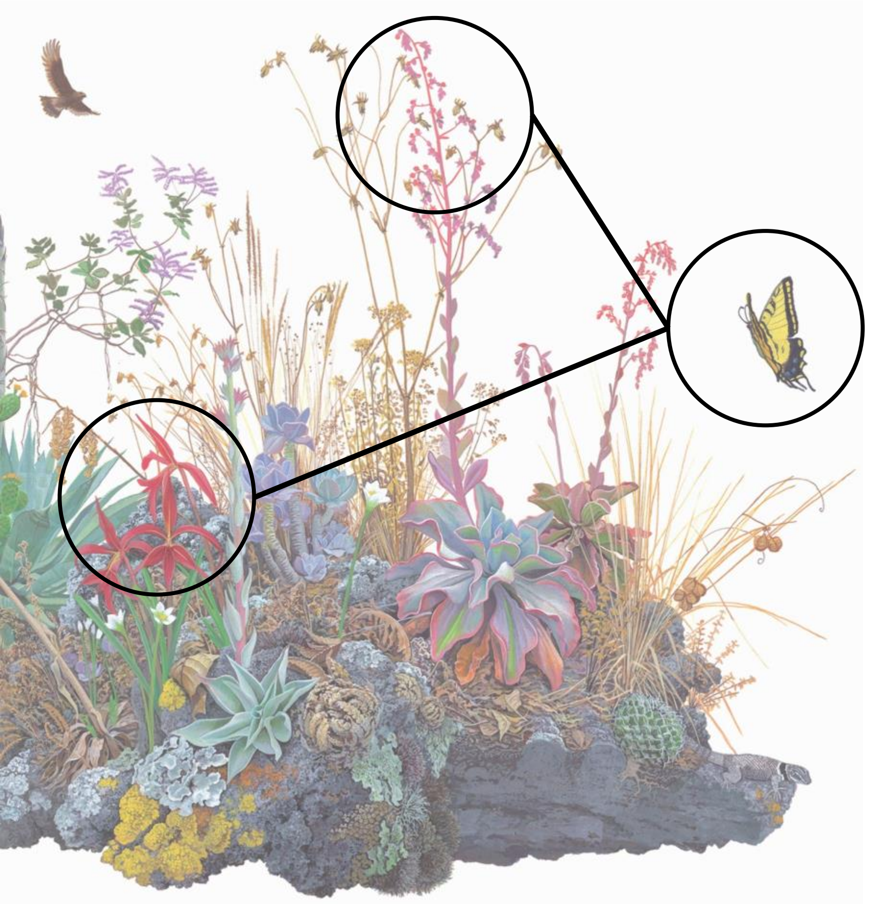


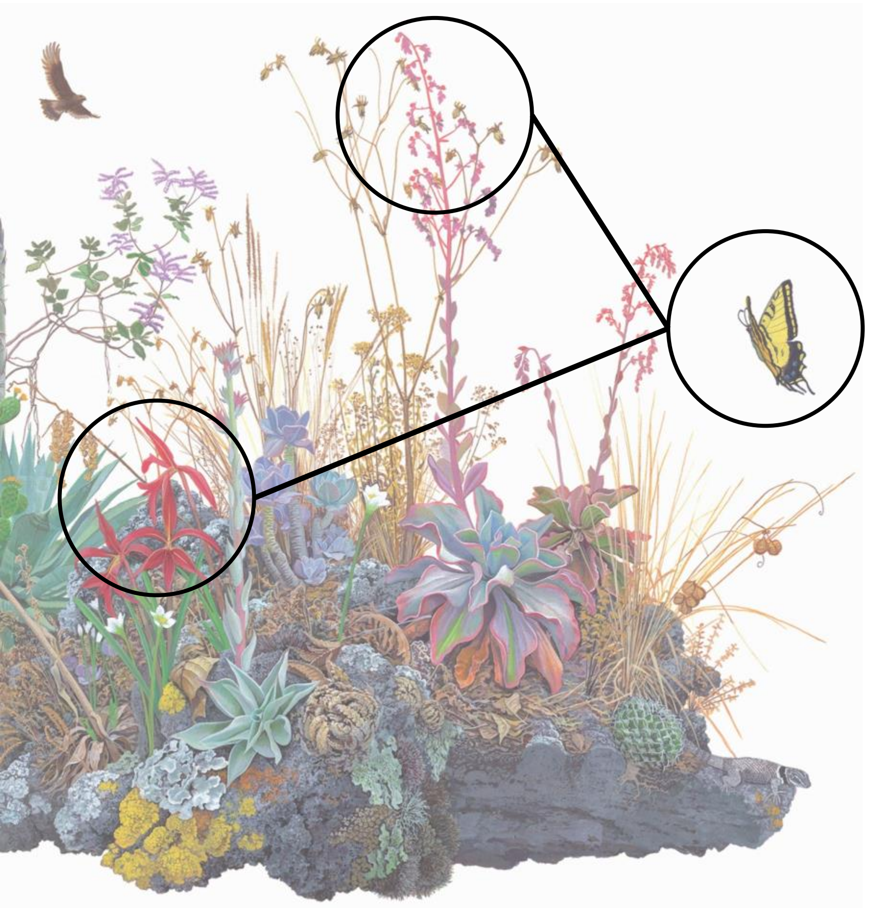
$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

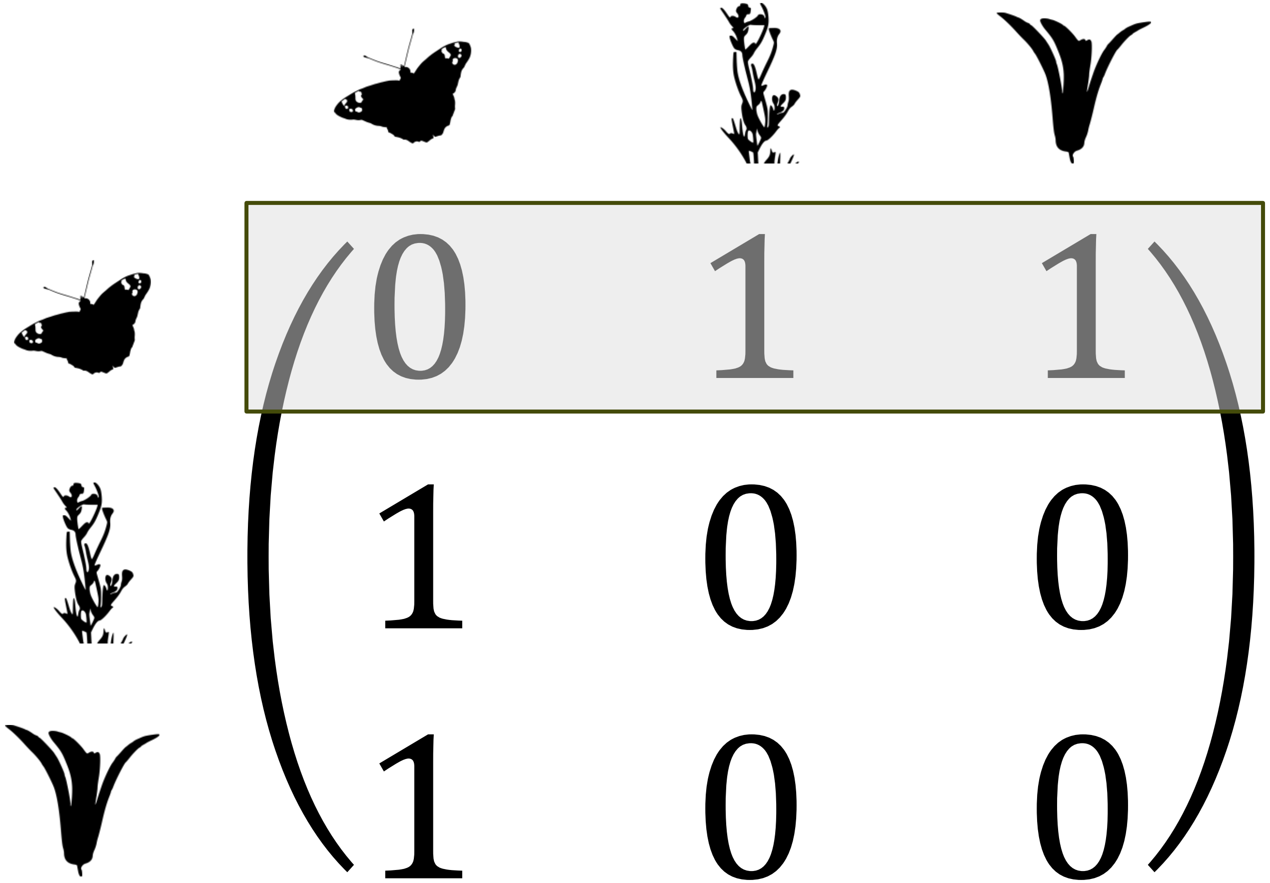
We can quantify degree from the interaction matrix



$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

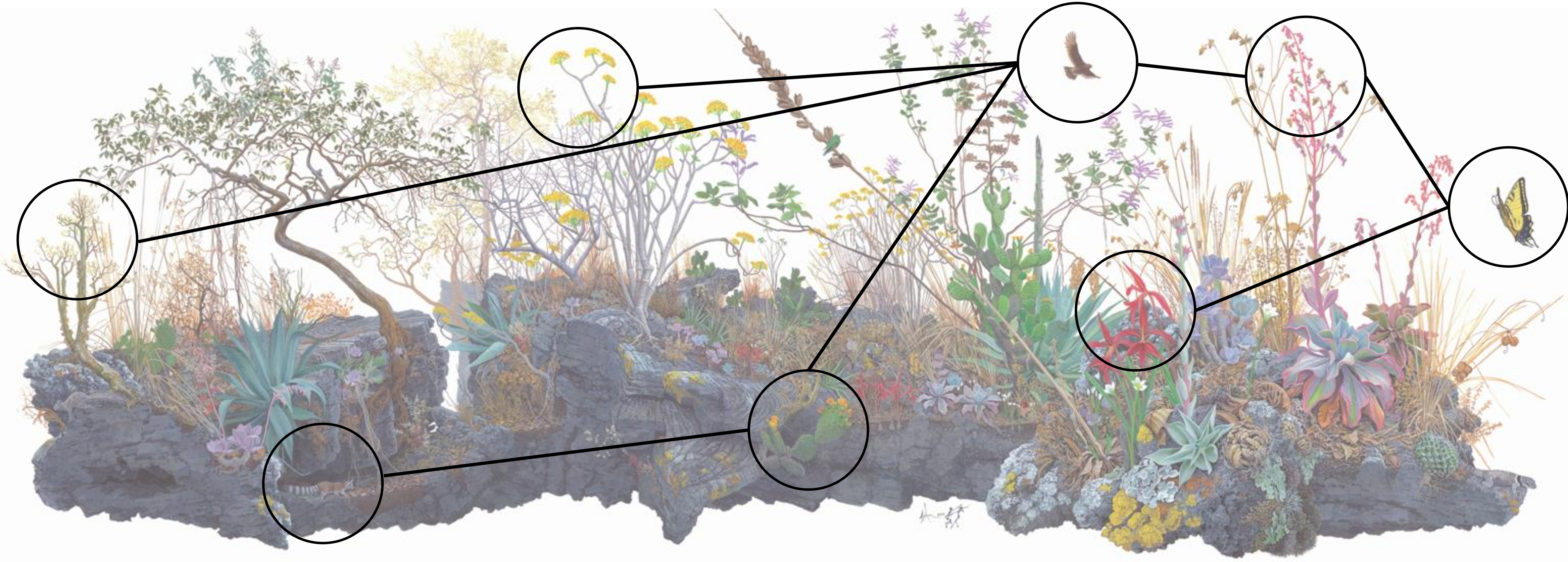
Matrix **A**, with entries $\boldsymbol{a_{ij}} = \mathbf{1}$ if species ***i*** interact with ***j***



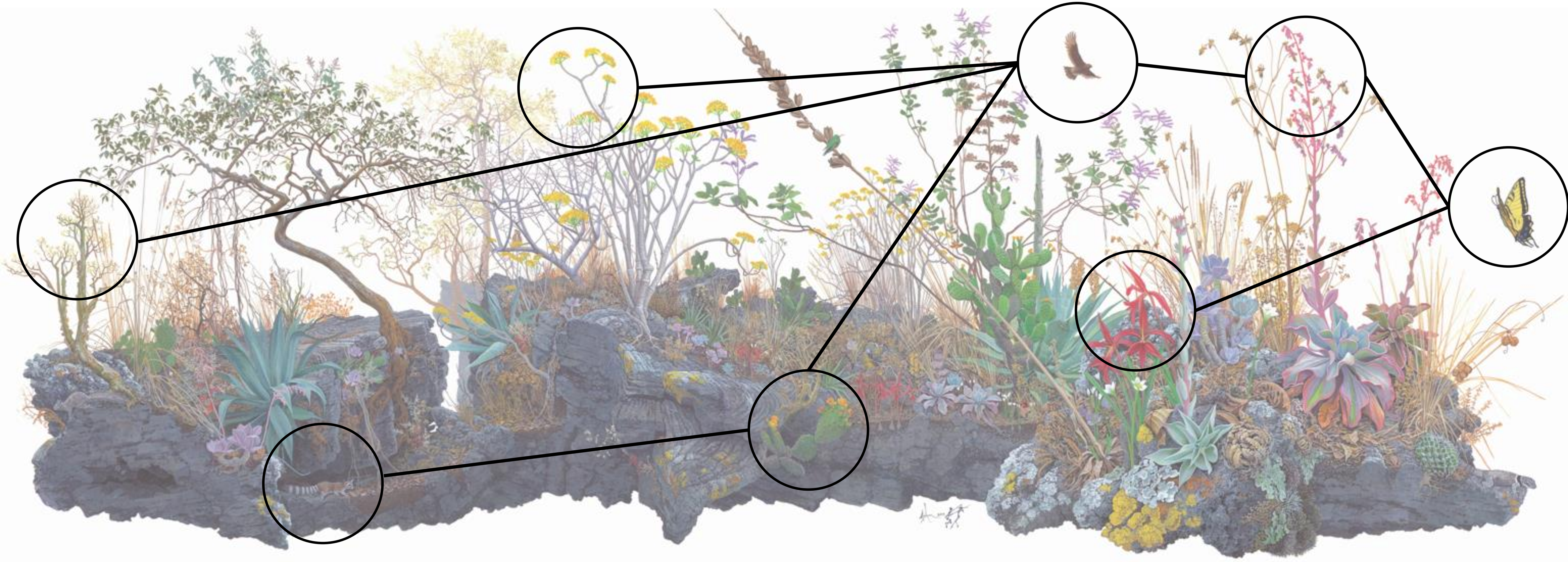


$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

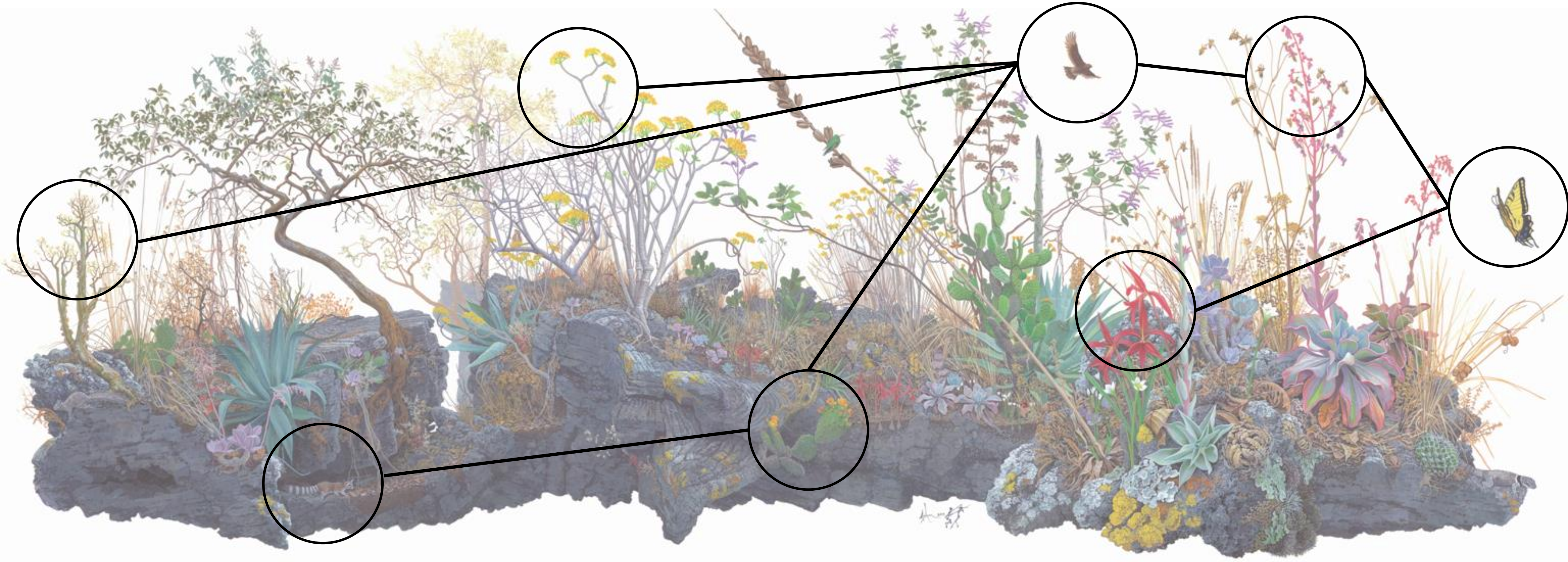
Degree = sum of the values within a row



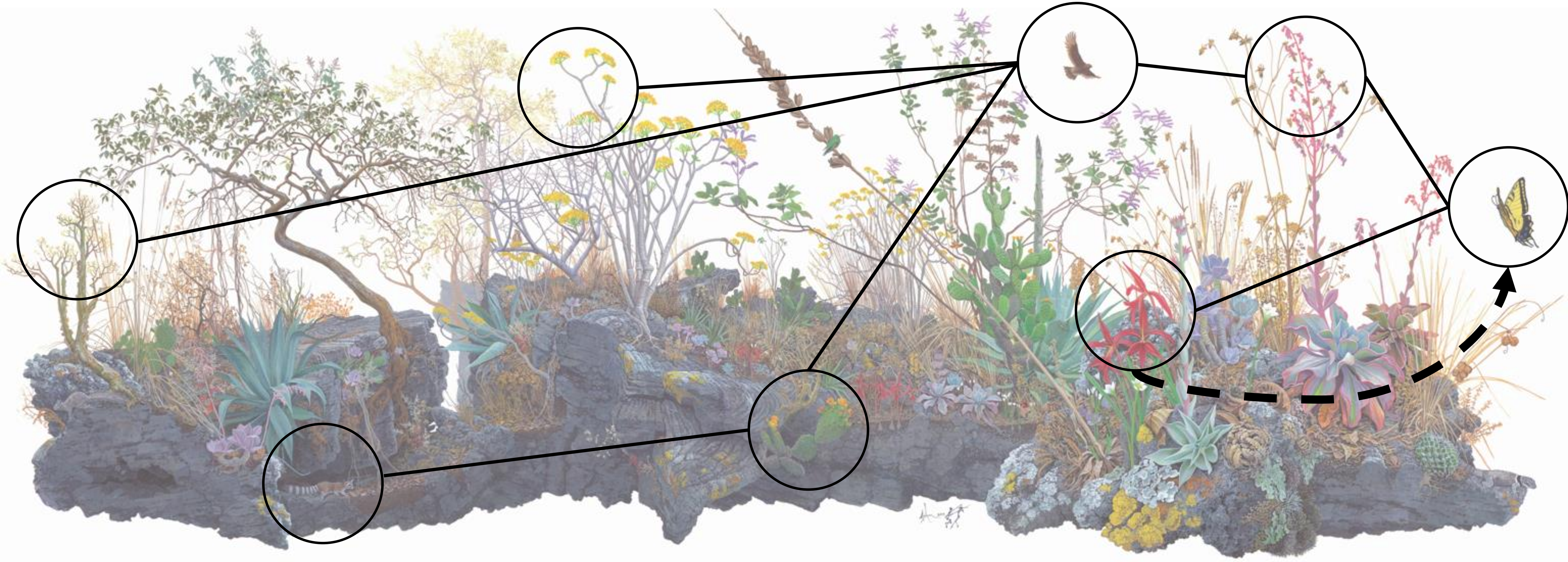
Degree is a measure of direct interactions



Main consequence of being part of a network?



Species can interact both directly and indirectly!



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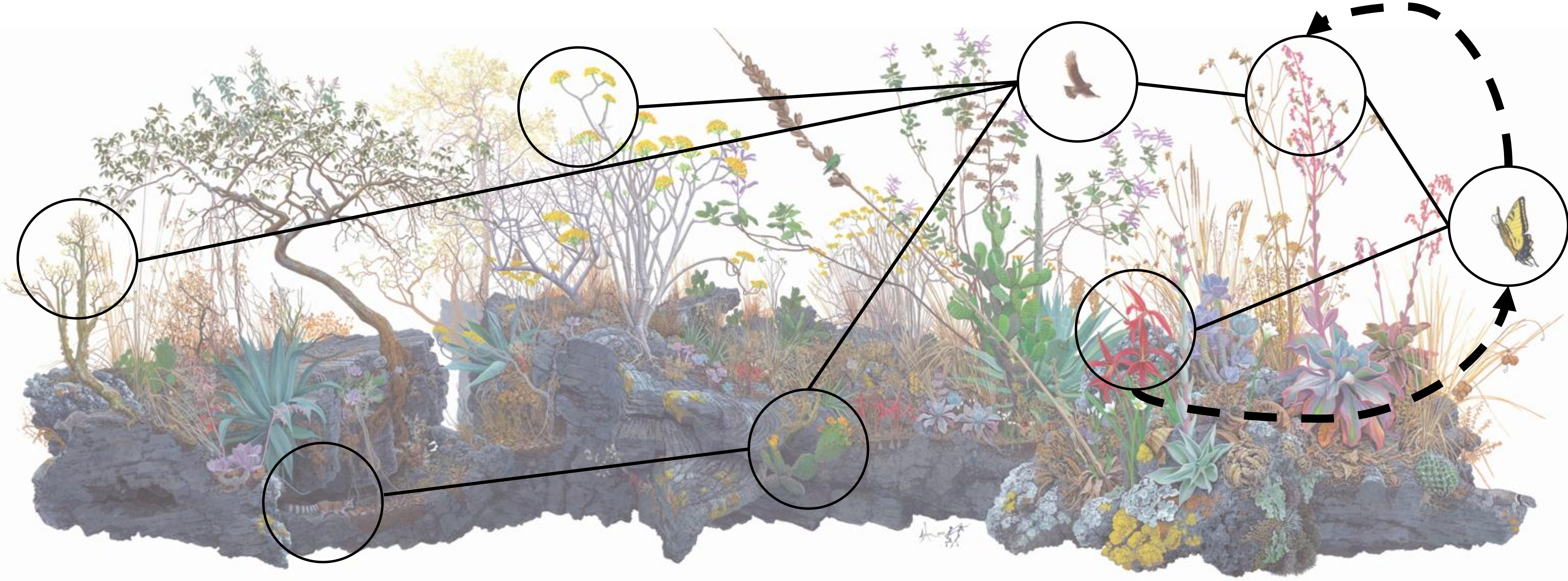

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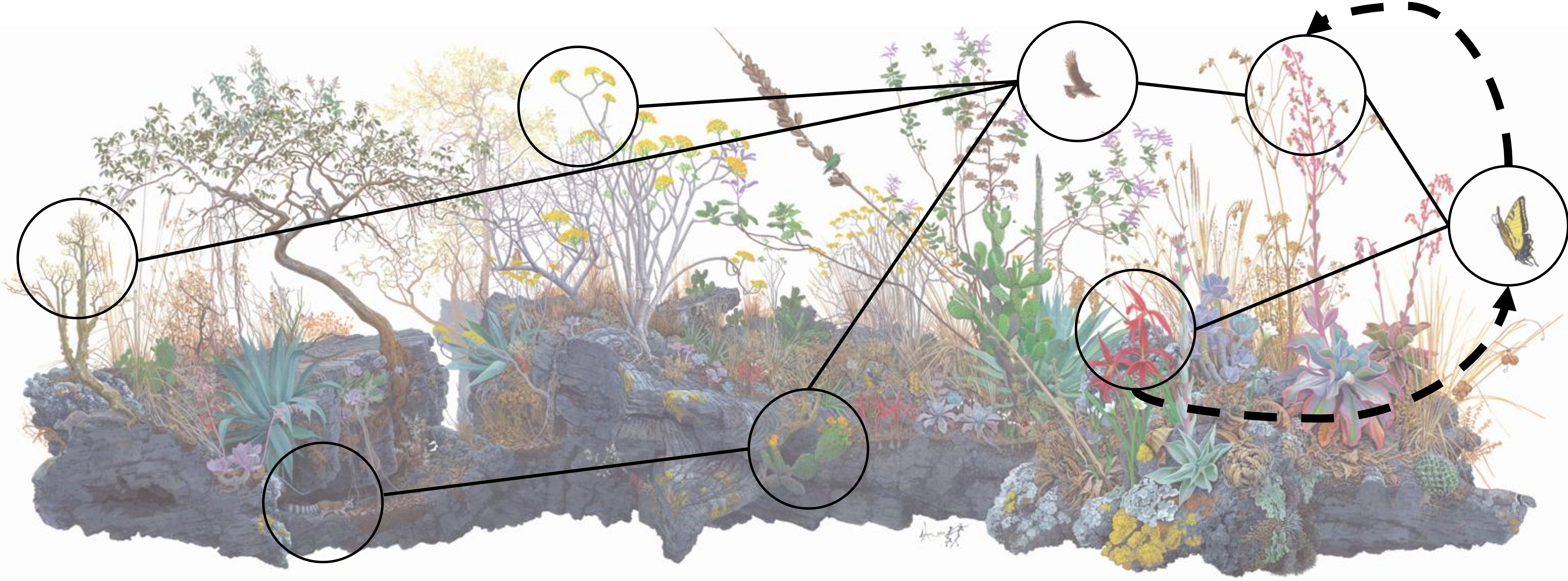
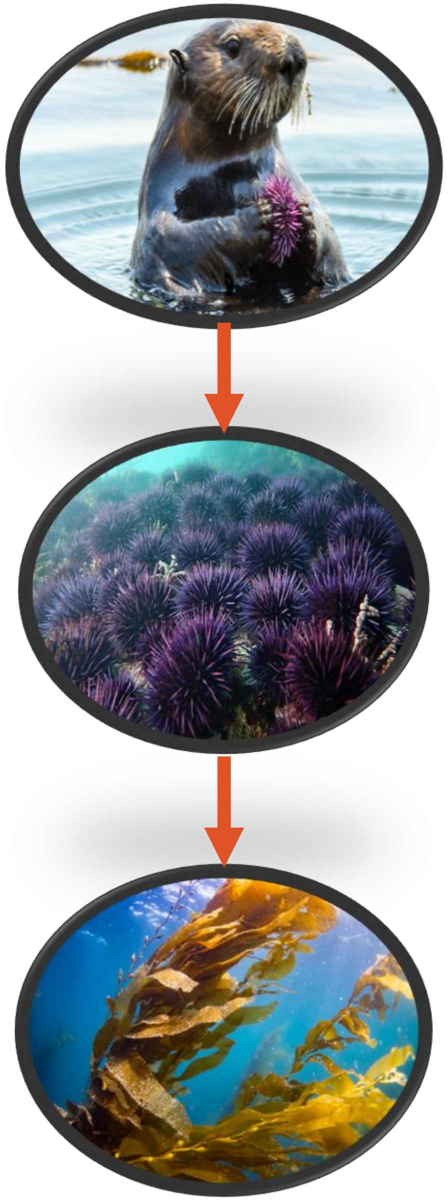
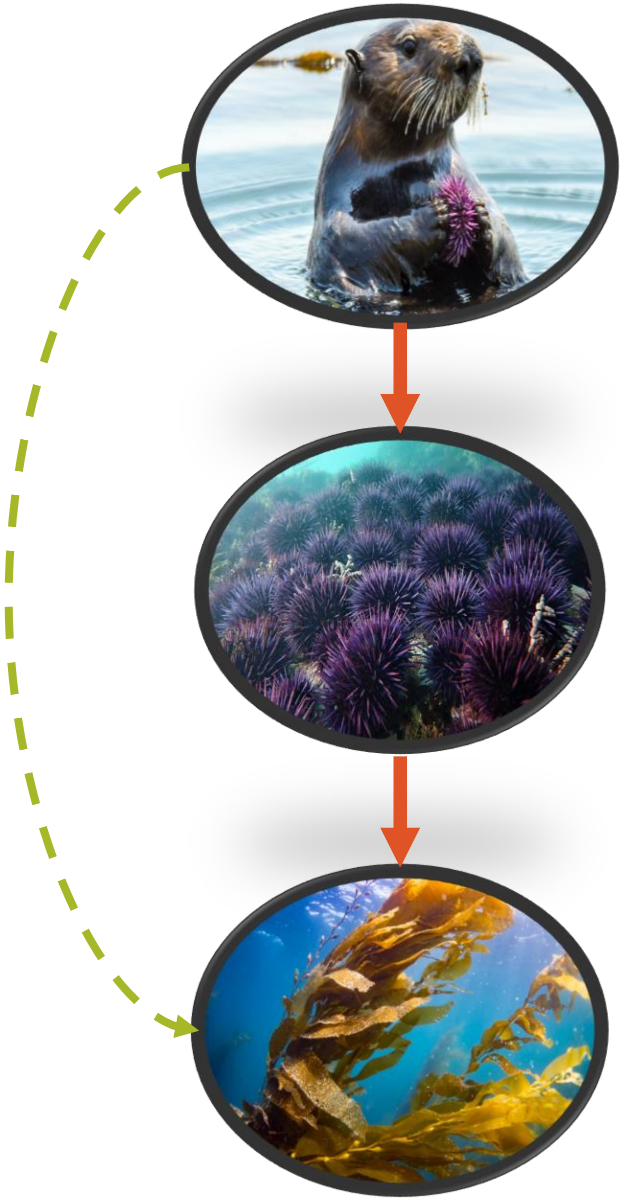
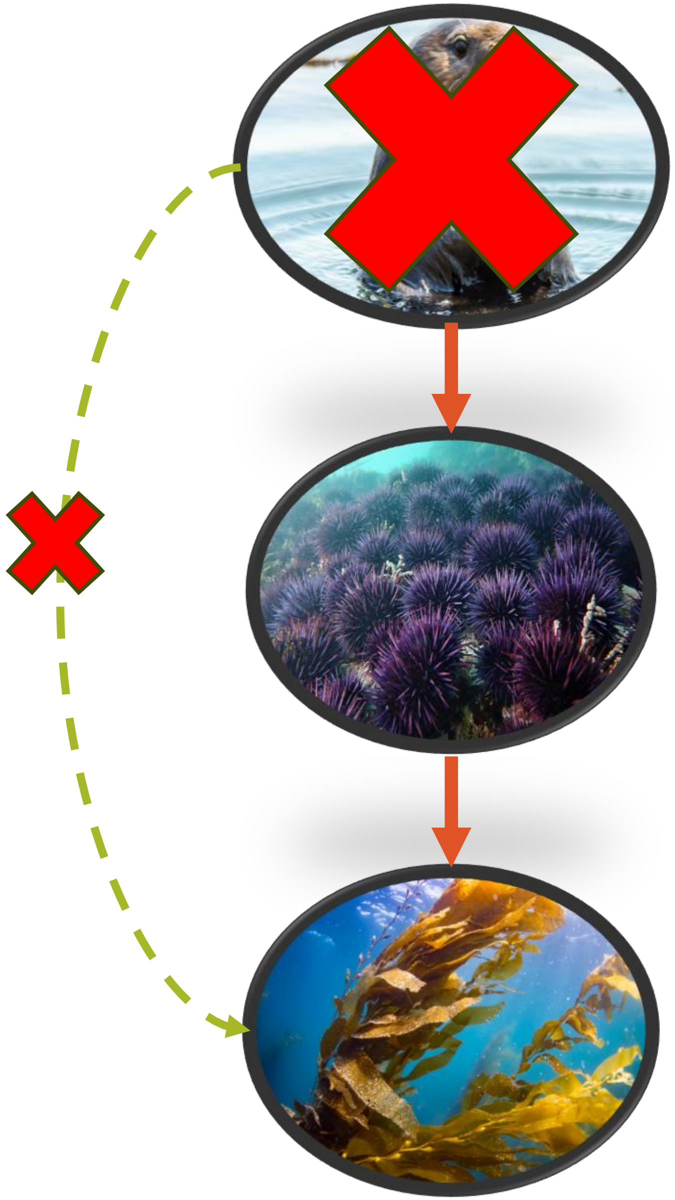
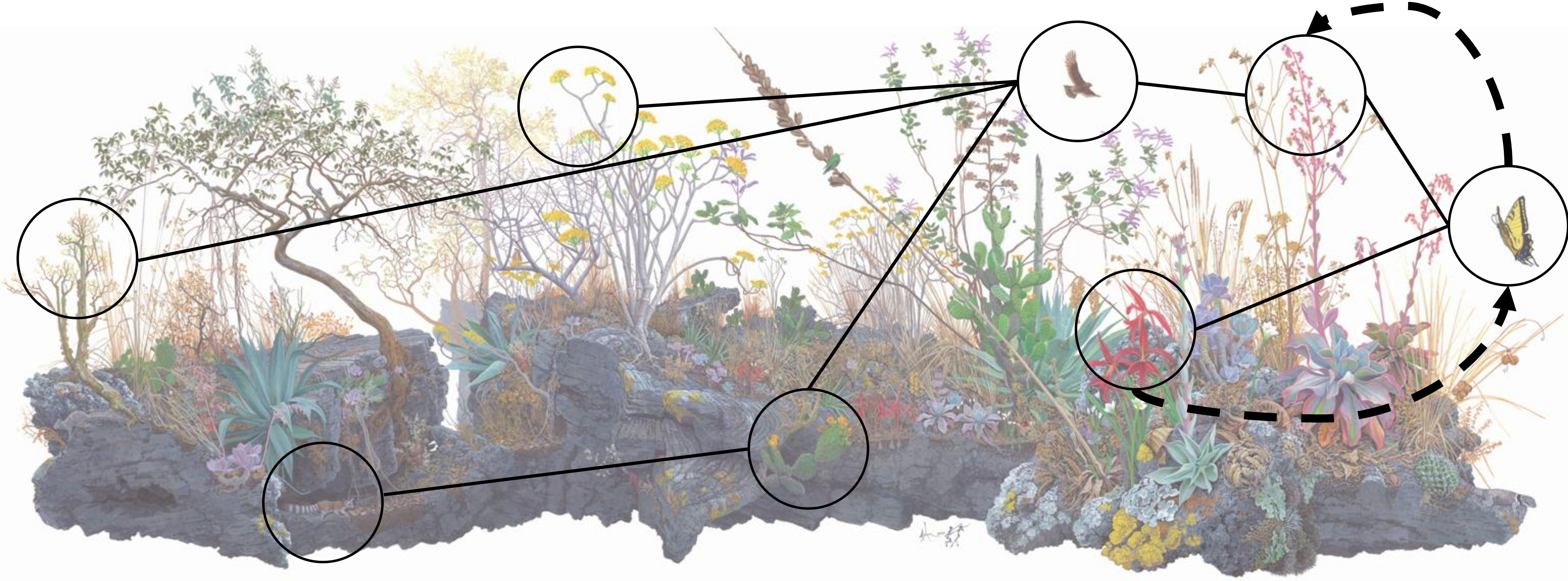


Do indirect interactions matter?

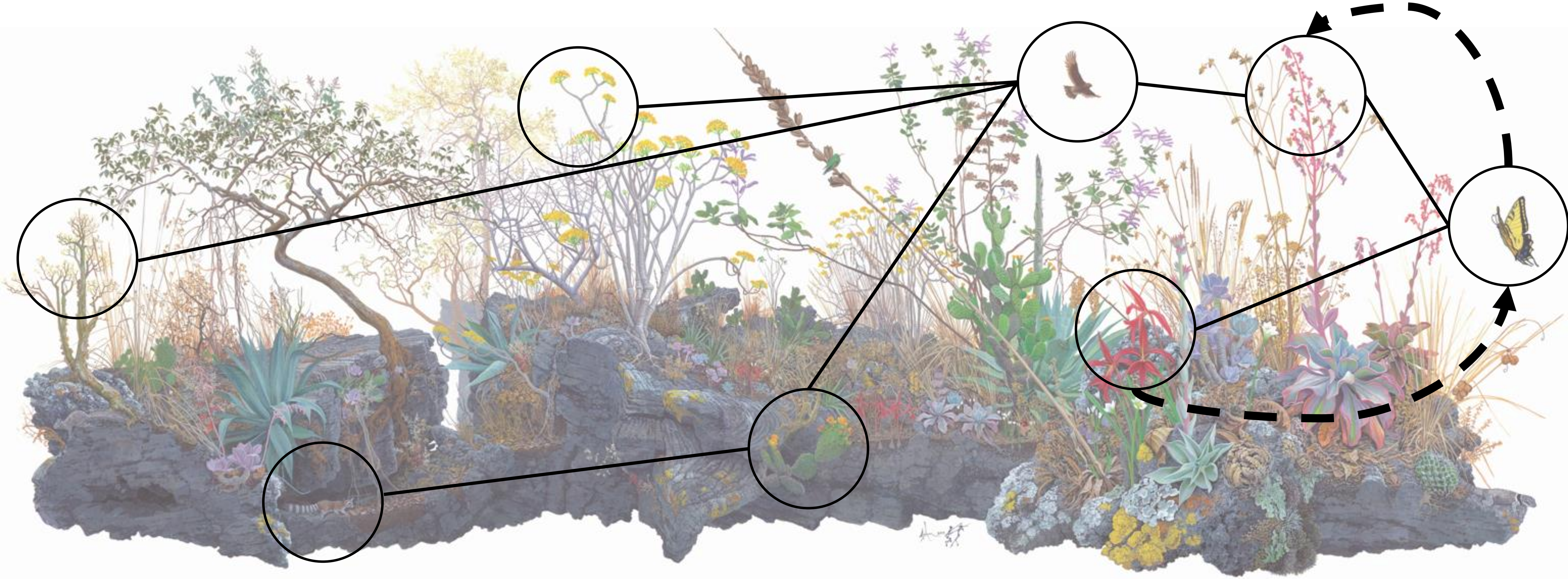


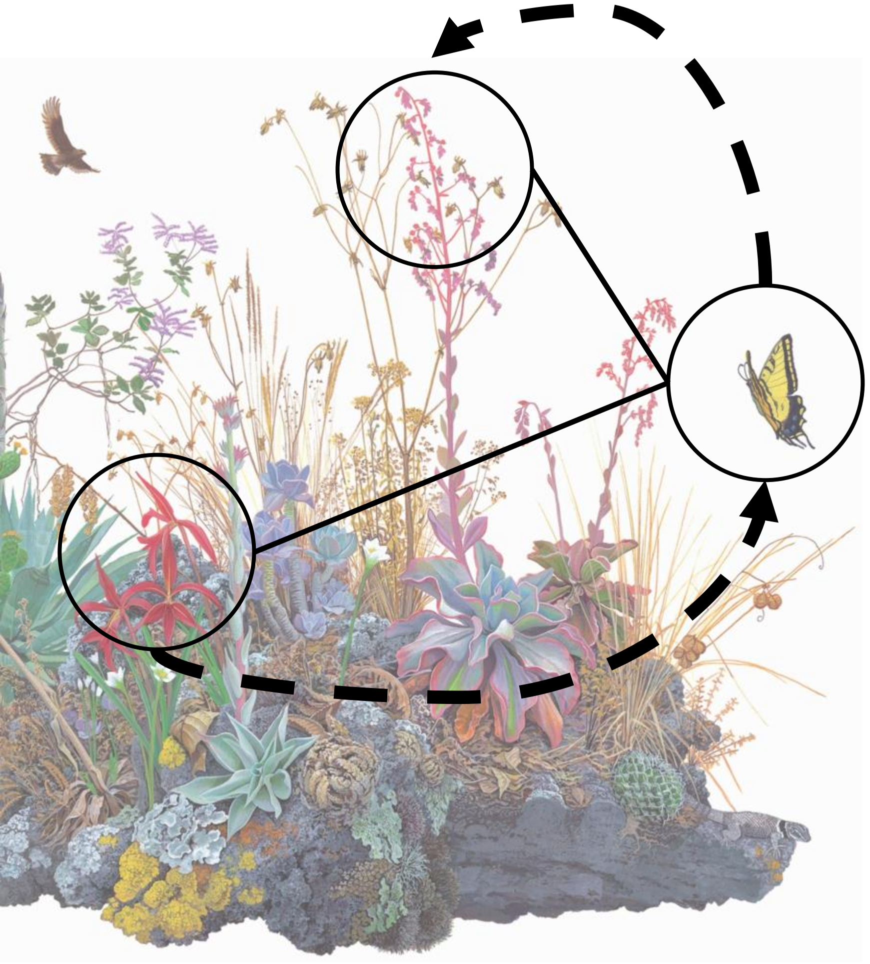





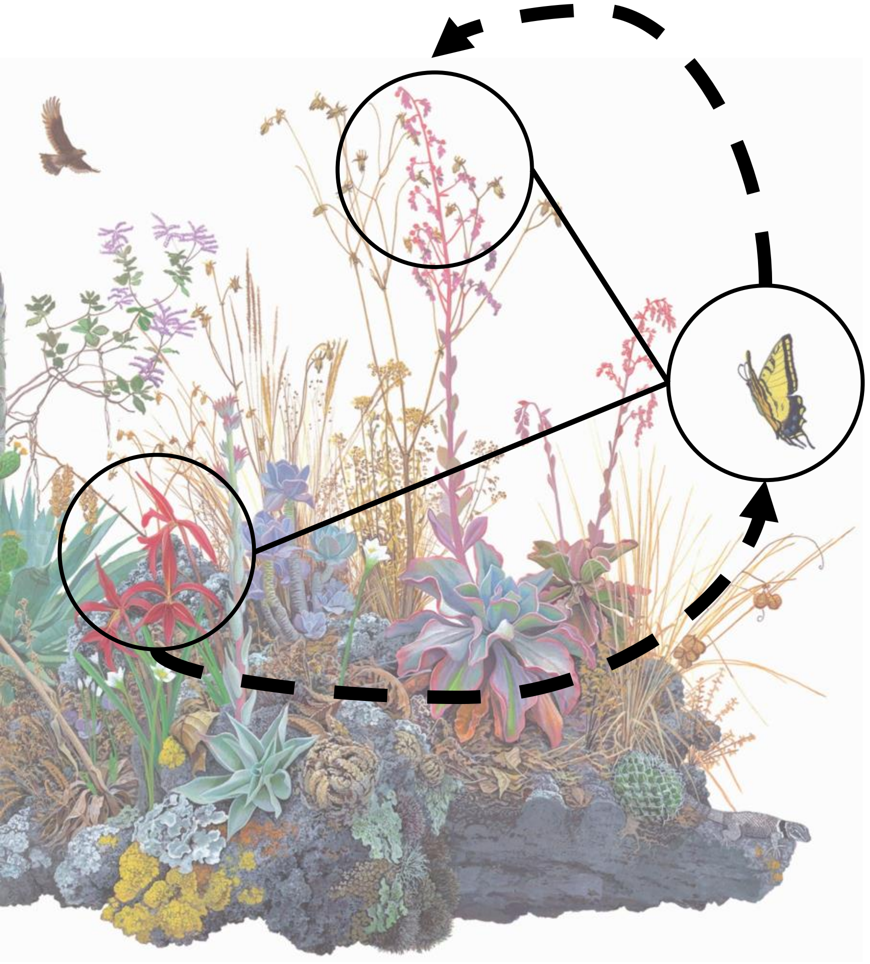
How can we quantify direct and indirect interactions?



Taking indirect interactions into account: shortest paths

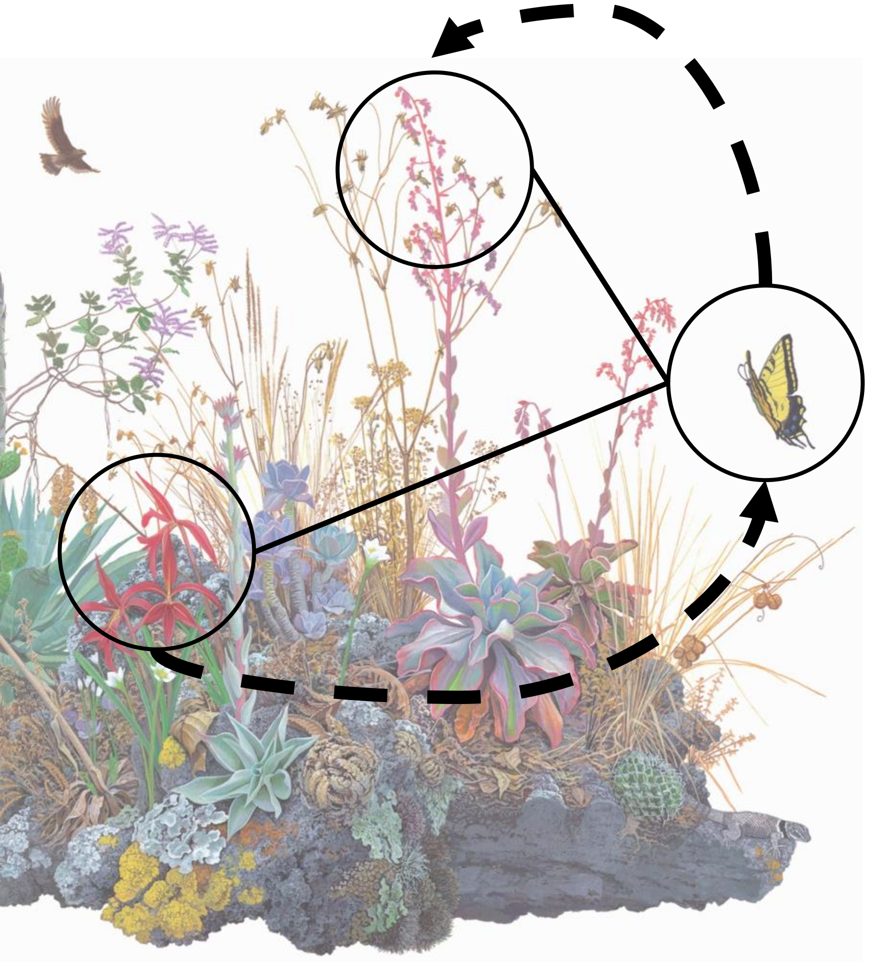


Distance matrix, **D**, with entries $\boldsymbol{d_{ij}}$ = shortest path length between species $\boldsymbol{i}$ and $\boldsymbol{j}$



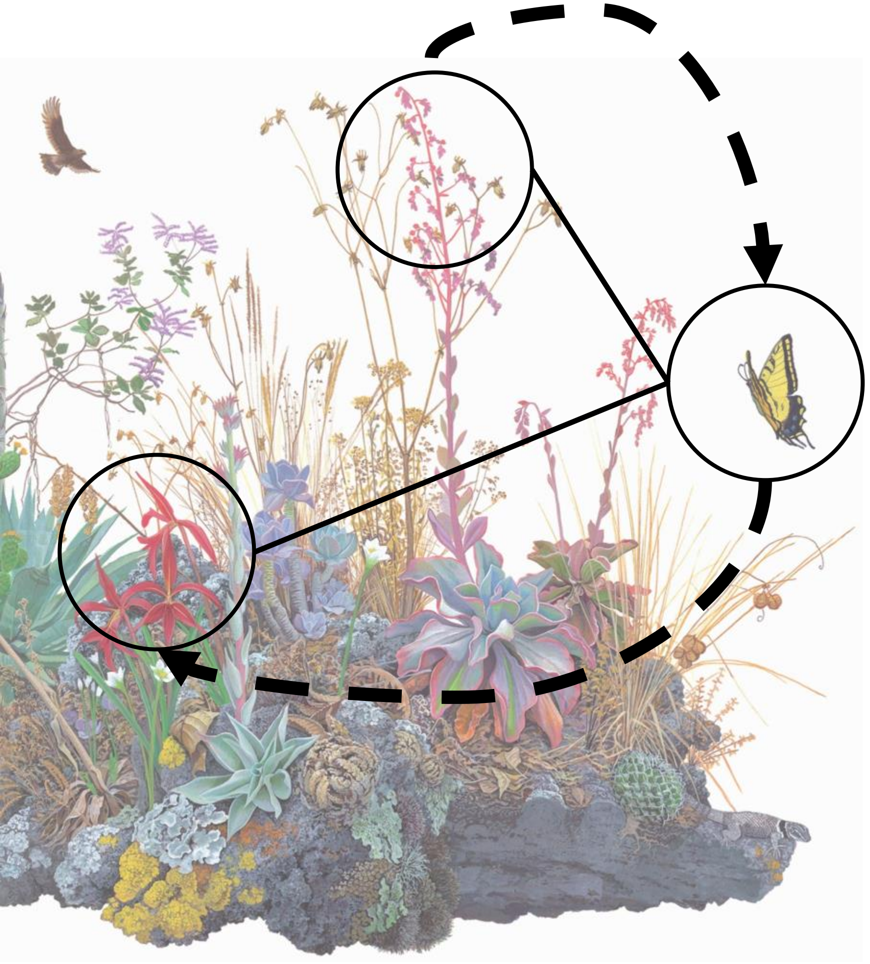
$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Distance matrix, **D**, with entries $\boldsymbol{d_{ij}}$ = shortest path length between species ***i*** and ***j***



$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 0 \end{pmatrix}$$

Distance matrix, **D**, with entries d_{ij} = shortest path length between species ***i*** and ***j***



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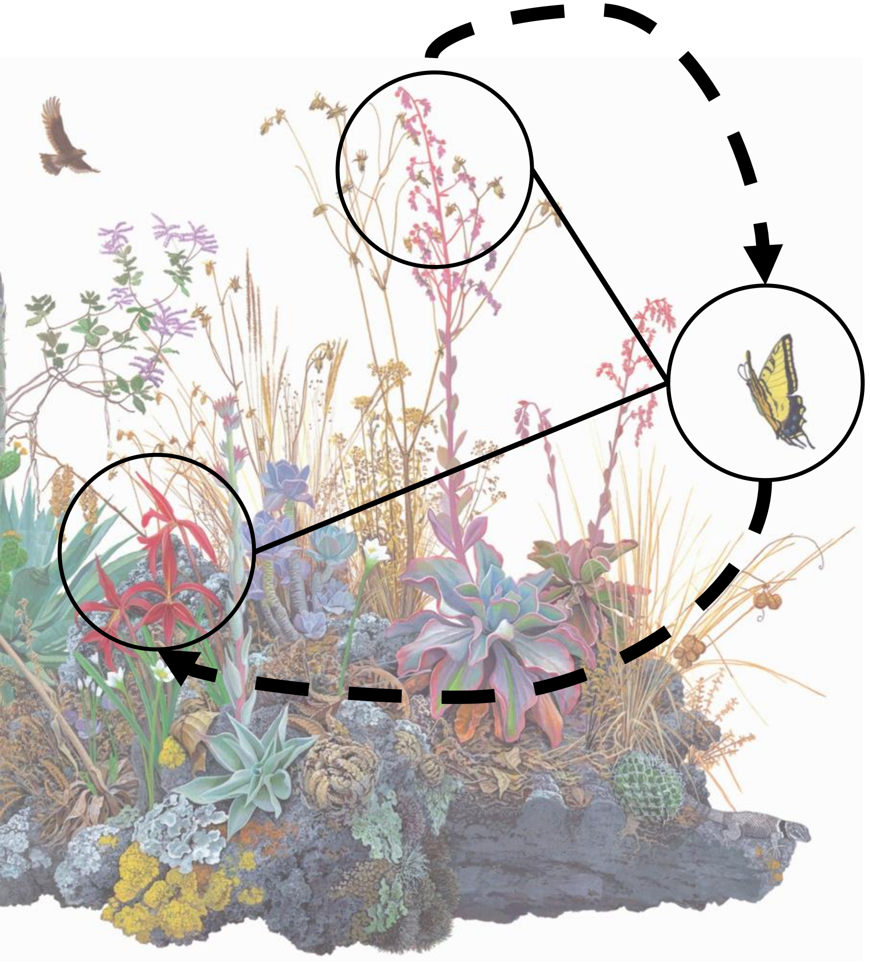
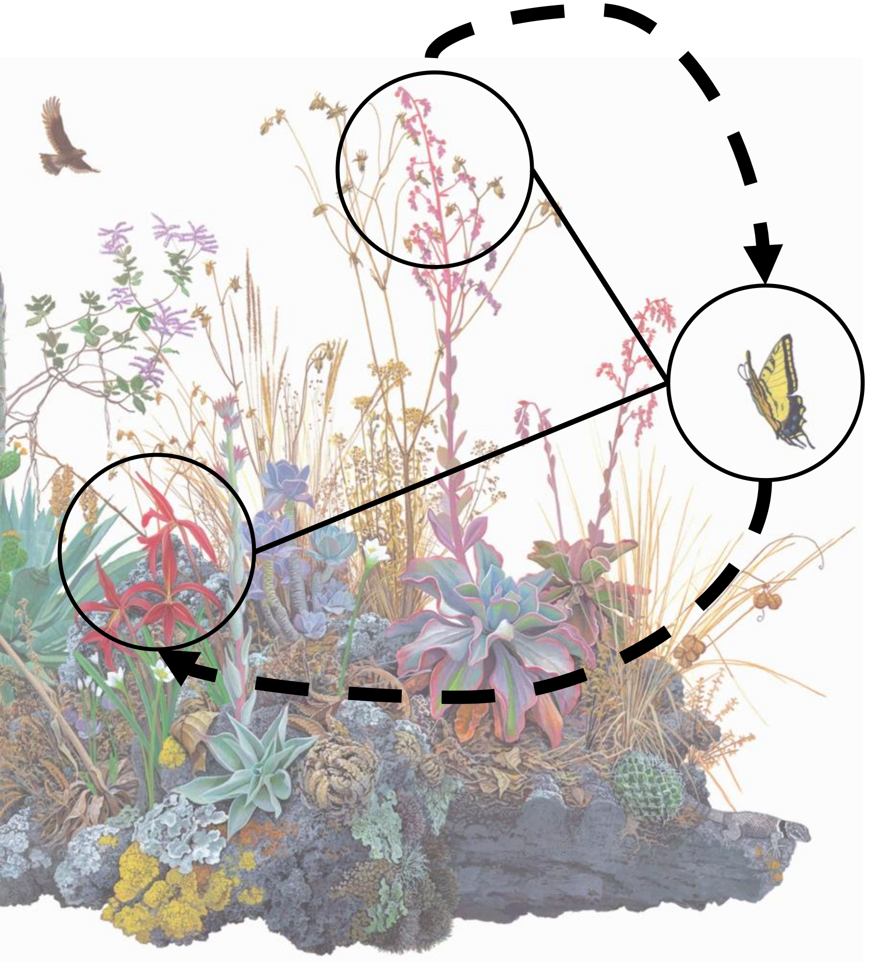
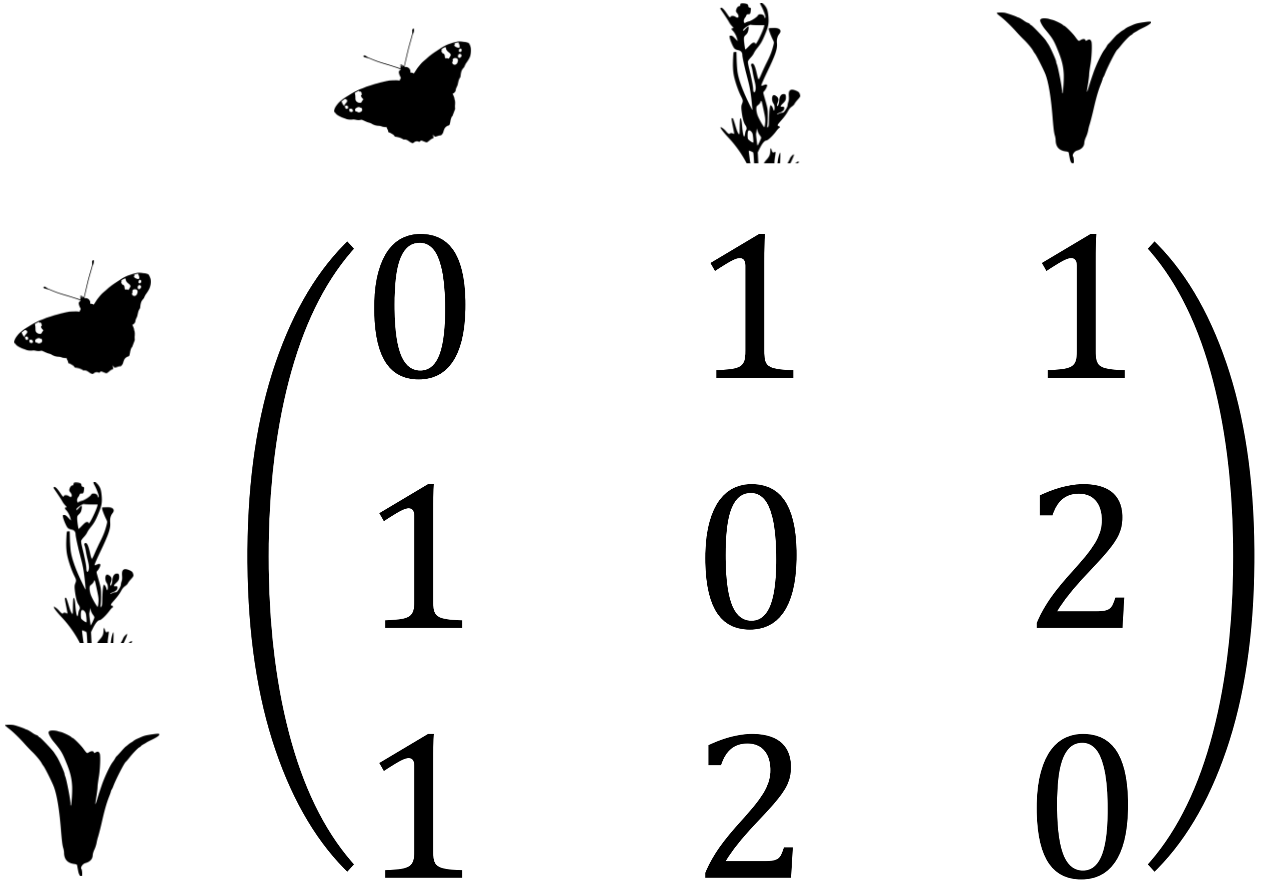


$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

How easily species can "access" others in the network?



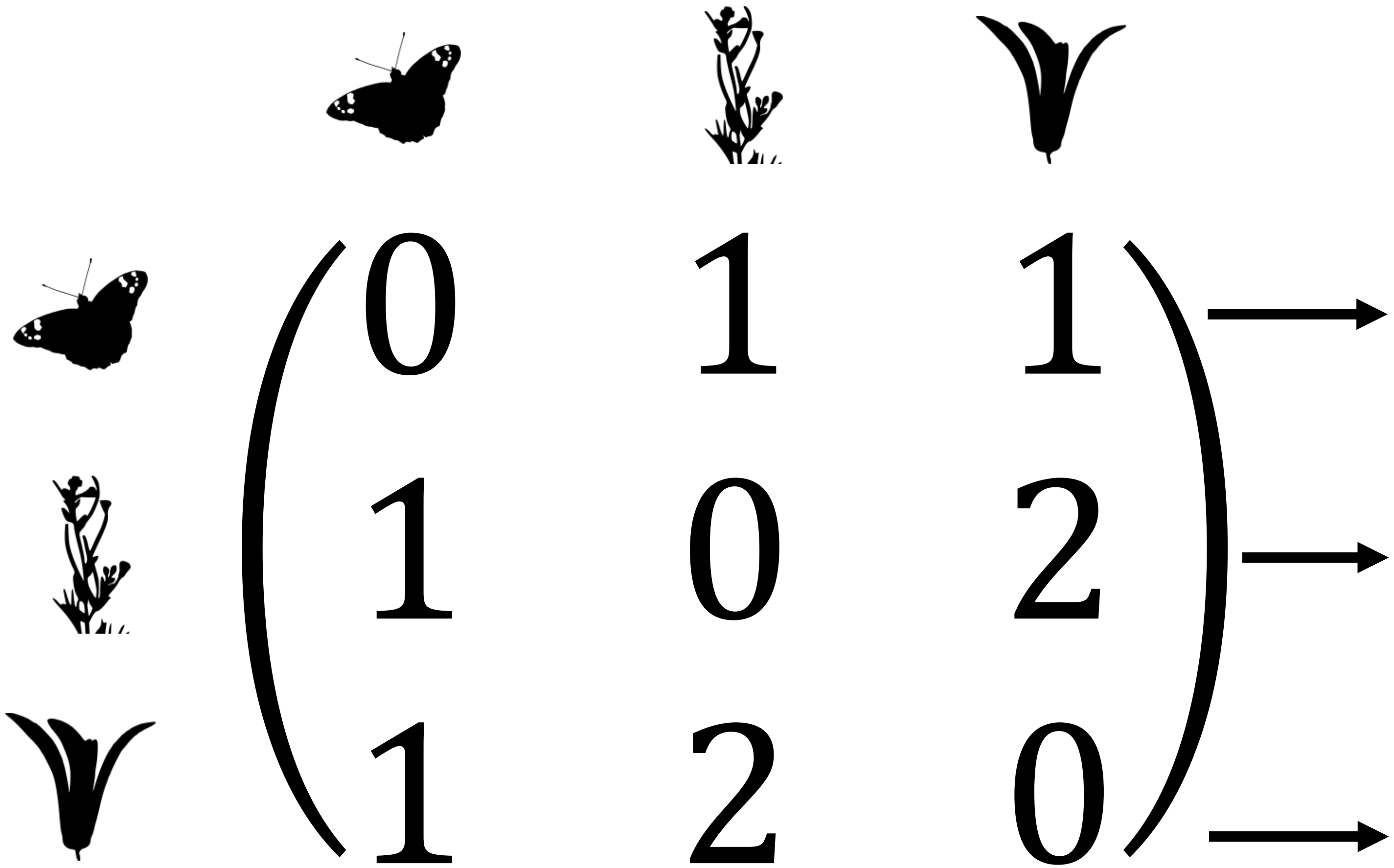


Taking the inverse of distance, $\frac{1}{d_{ij}}$, and computing the average

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix} \longrightarrow \left(\frac{1}{1} + \frac{1}{1}\right) = 2$$

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix} \longrightarrow \left(\frac{1}{1} + \frac{1}{1}\right) = 2 \rightarrow \frac{2}{2} = 1$$



$$\left(\frac{1}{1}+\frac{1}{1}\right) = 2 \rightarrow \frac{2}{2} = 1$$

$$\left(\frac{1}{1}+\frac{1}{2}\right) = 1.5 \rightarrow \frac{1.5}{2} = 0.75$$

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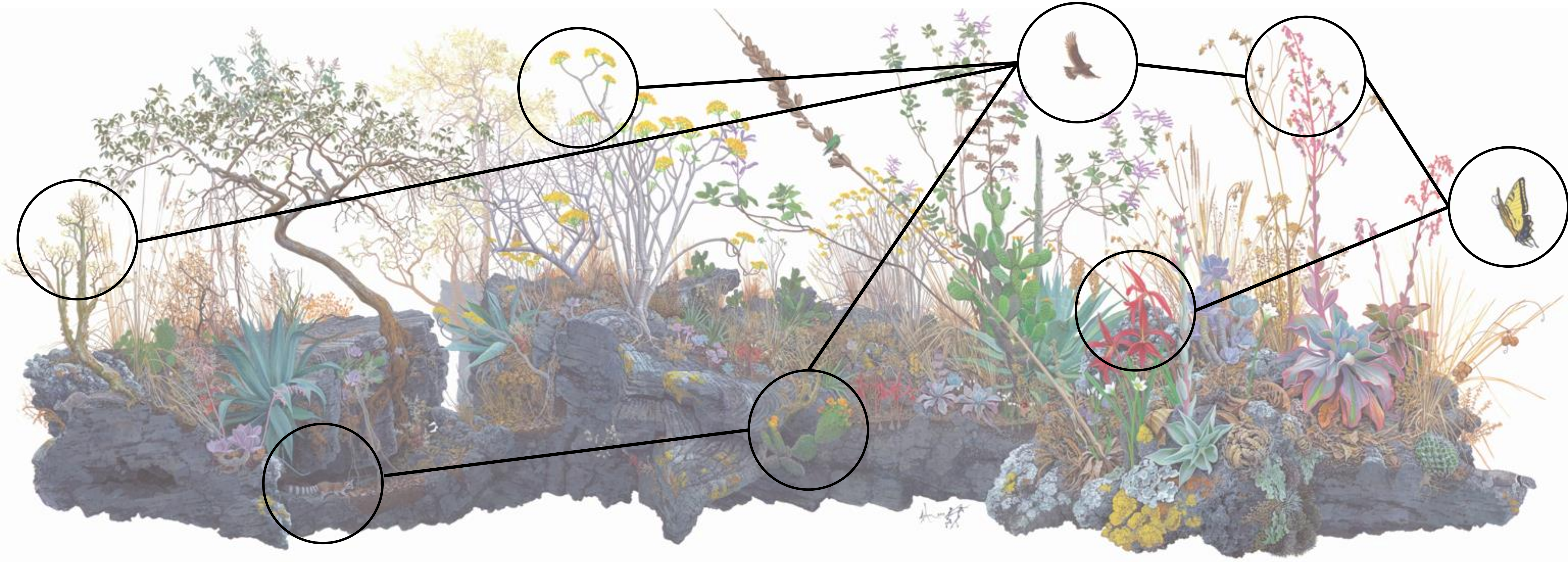
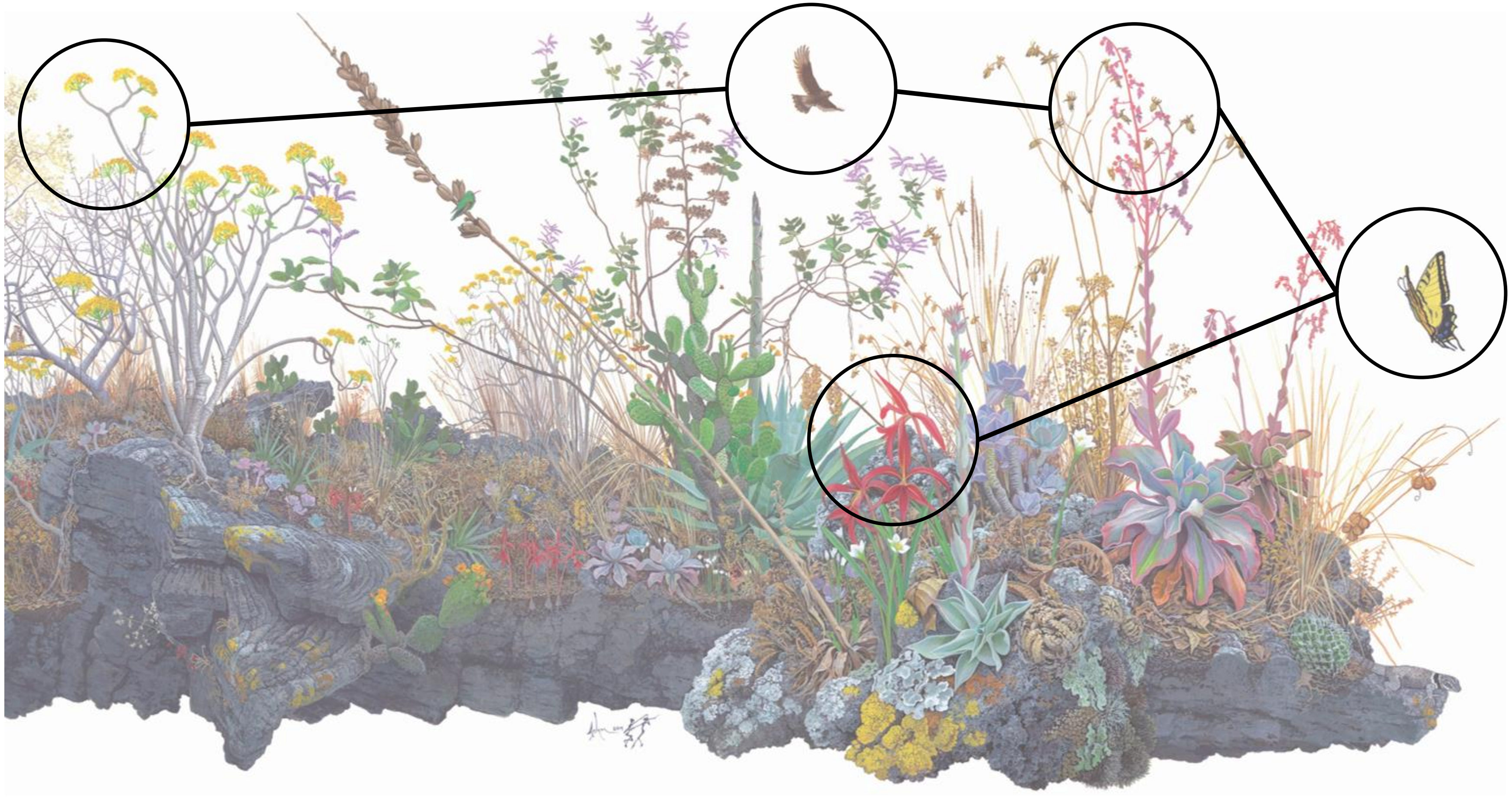
A general equation: $\boldsymbol{C_i} = \dfrac{\left(\sum_{j \neq i}^{N} \frac{1}{d_{ij}}\right)}{N-1}$



Are shortest paths the only ones that matter?



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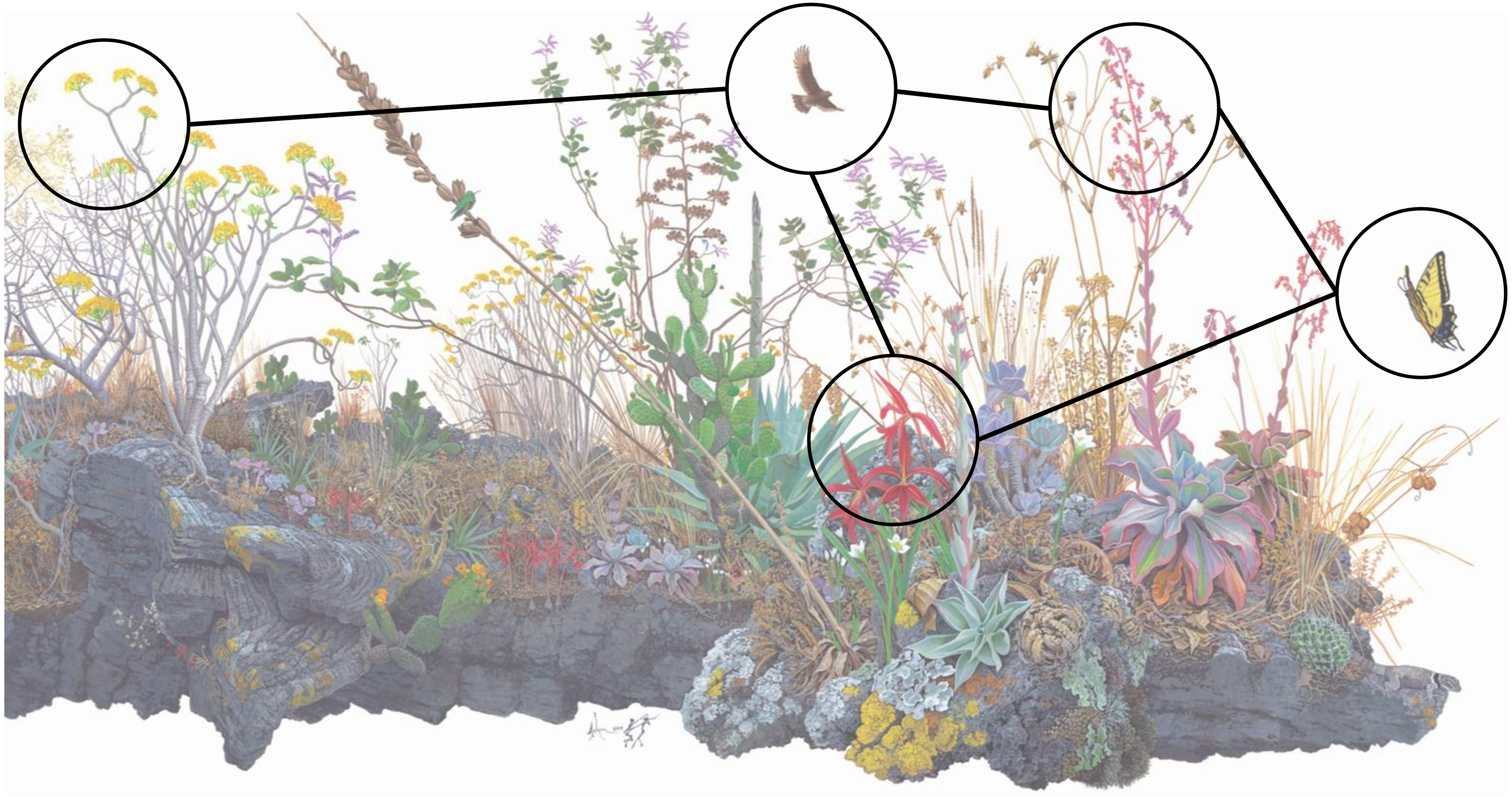

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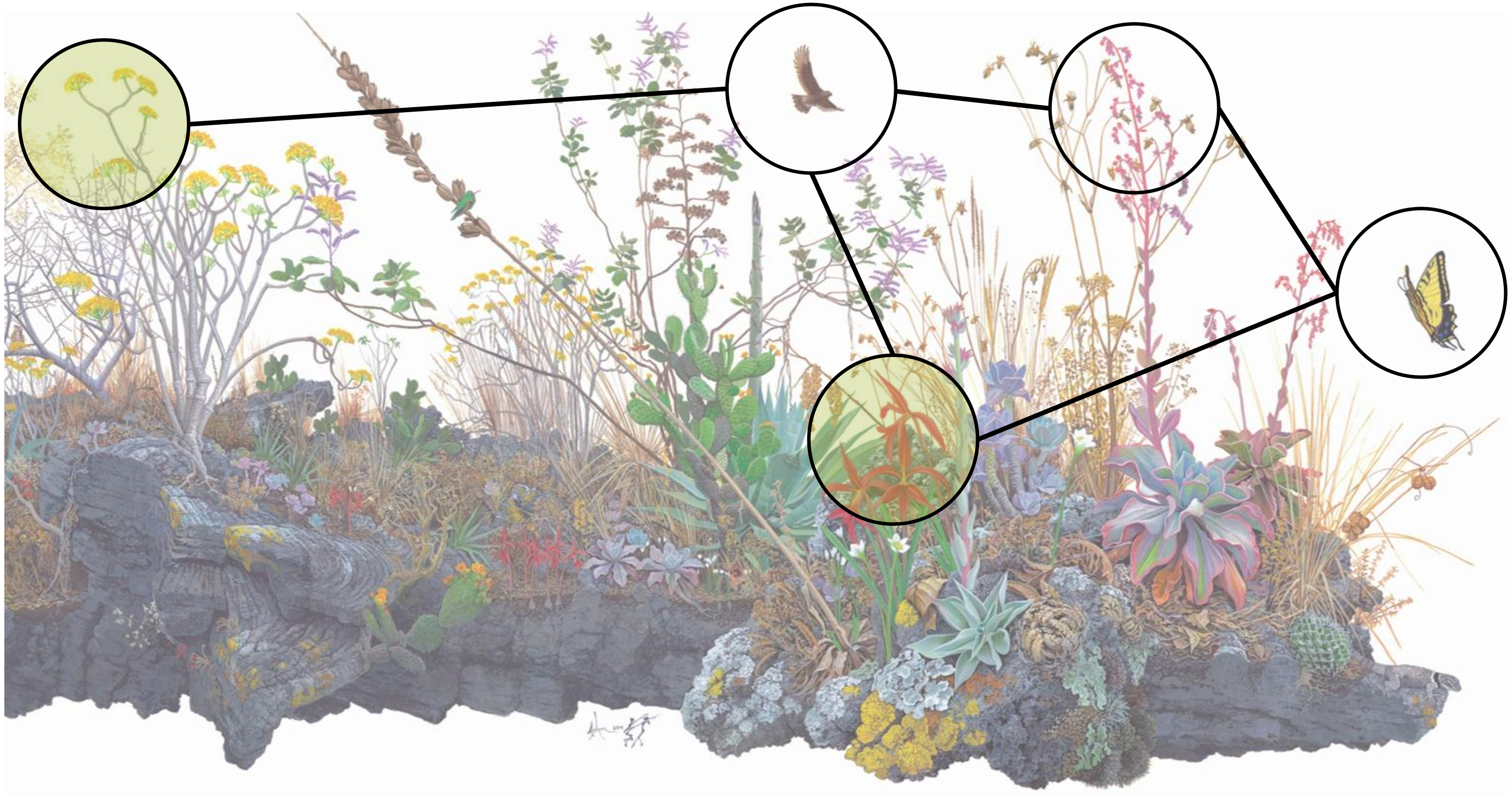

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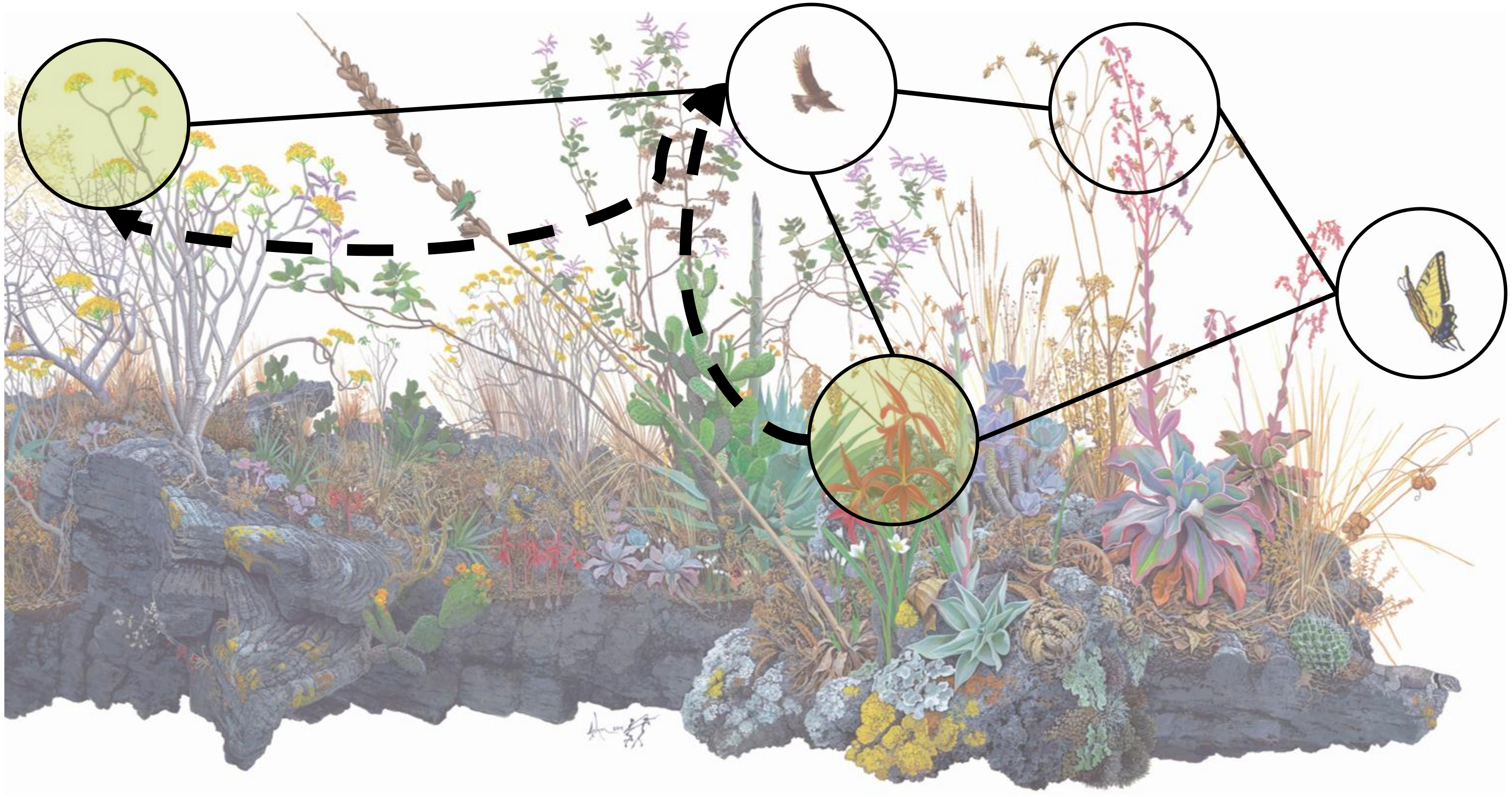
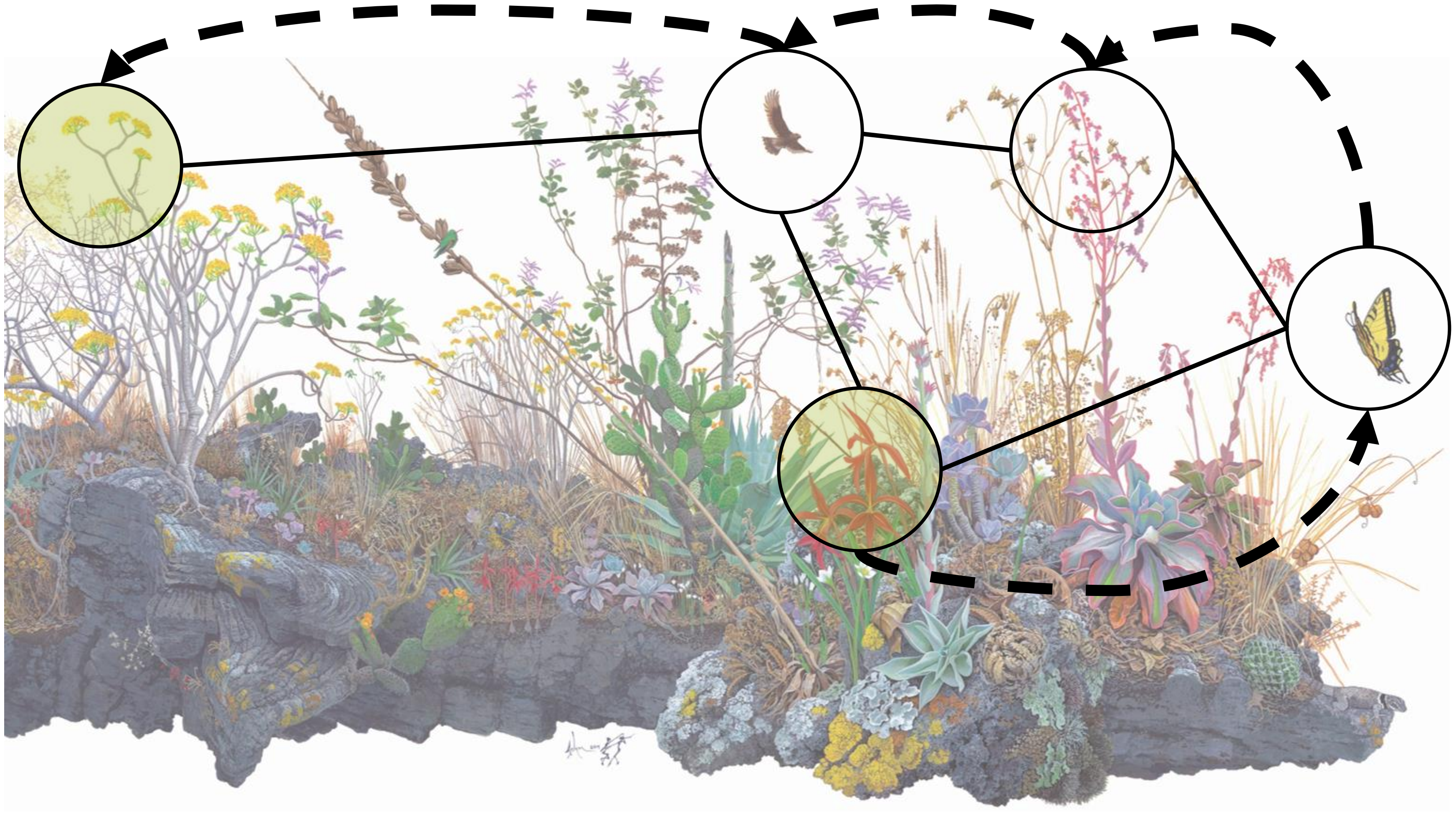
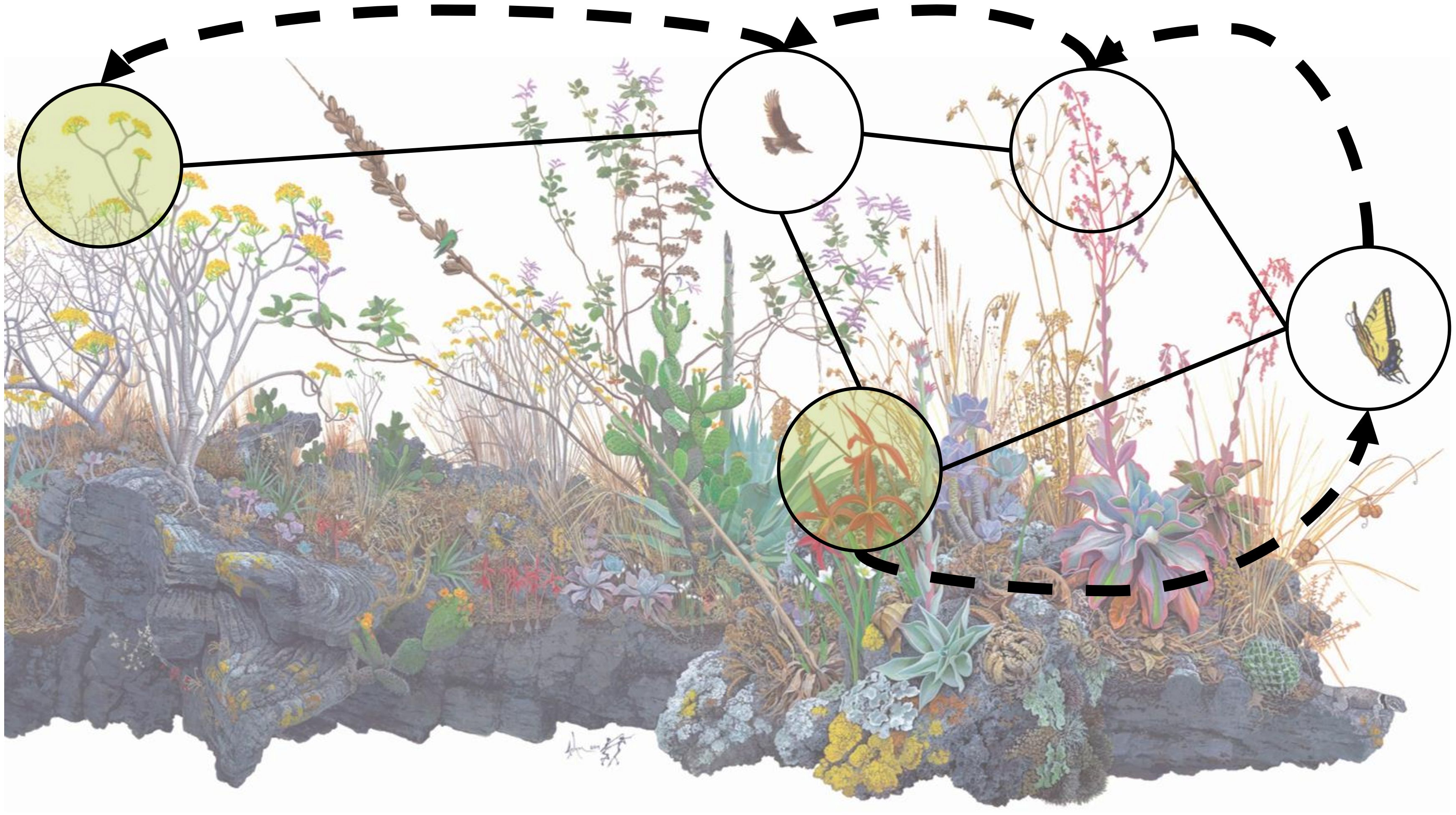


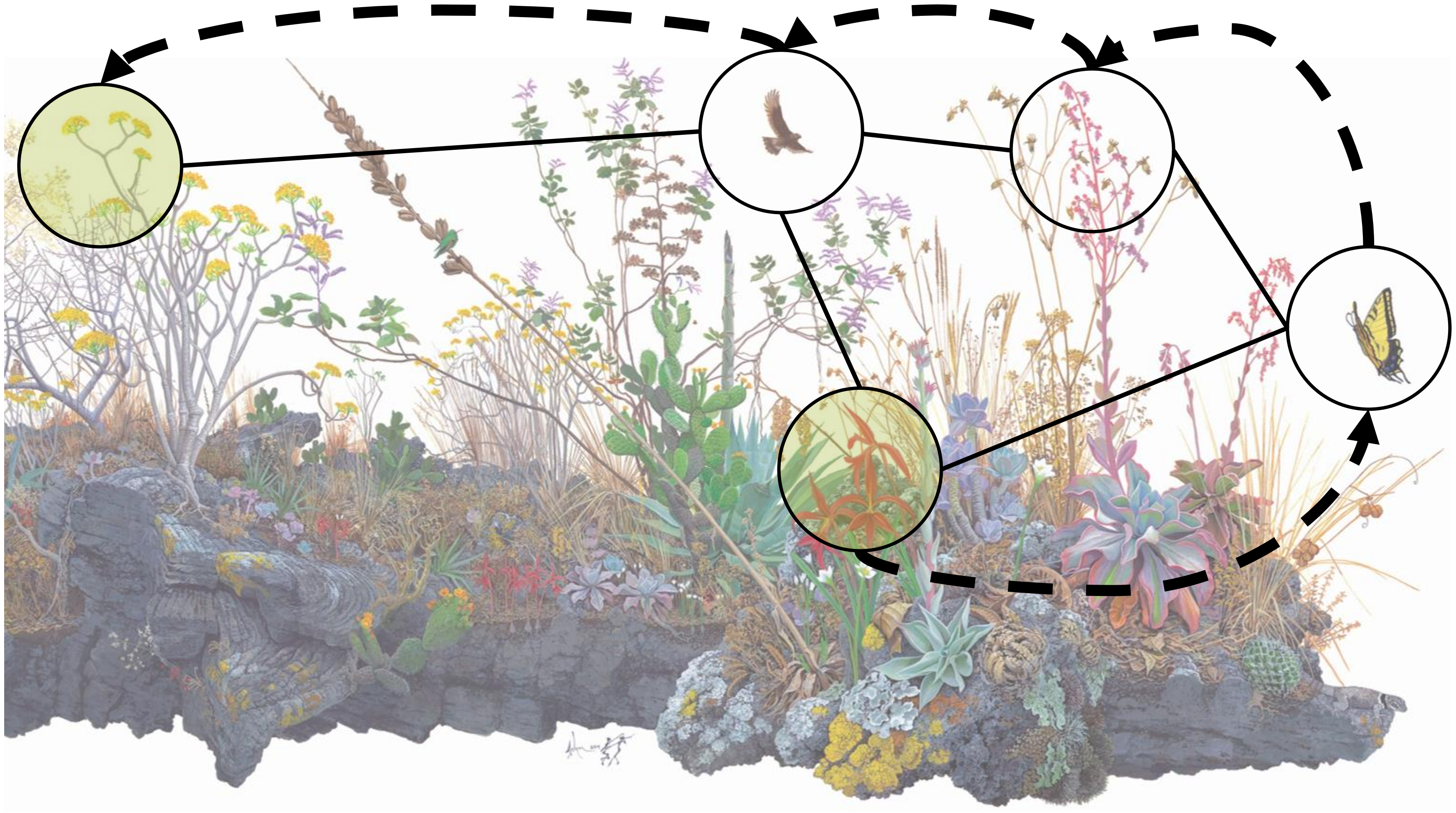
Shortest path: length 2



Alternative, longer pathway: length 4

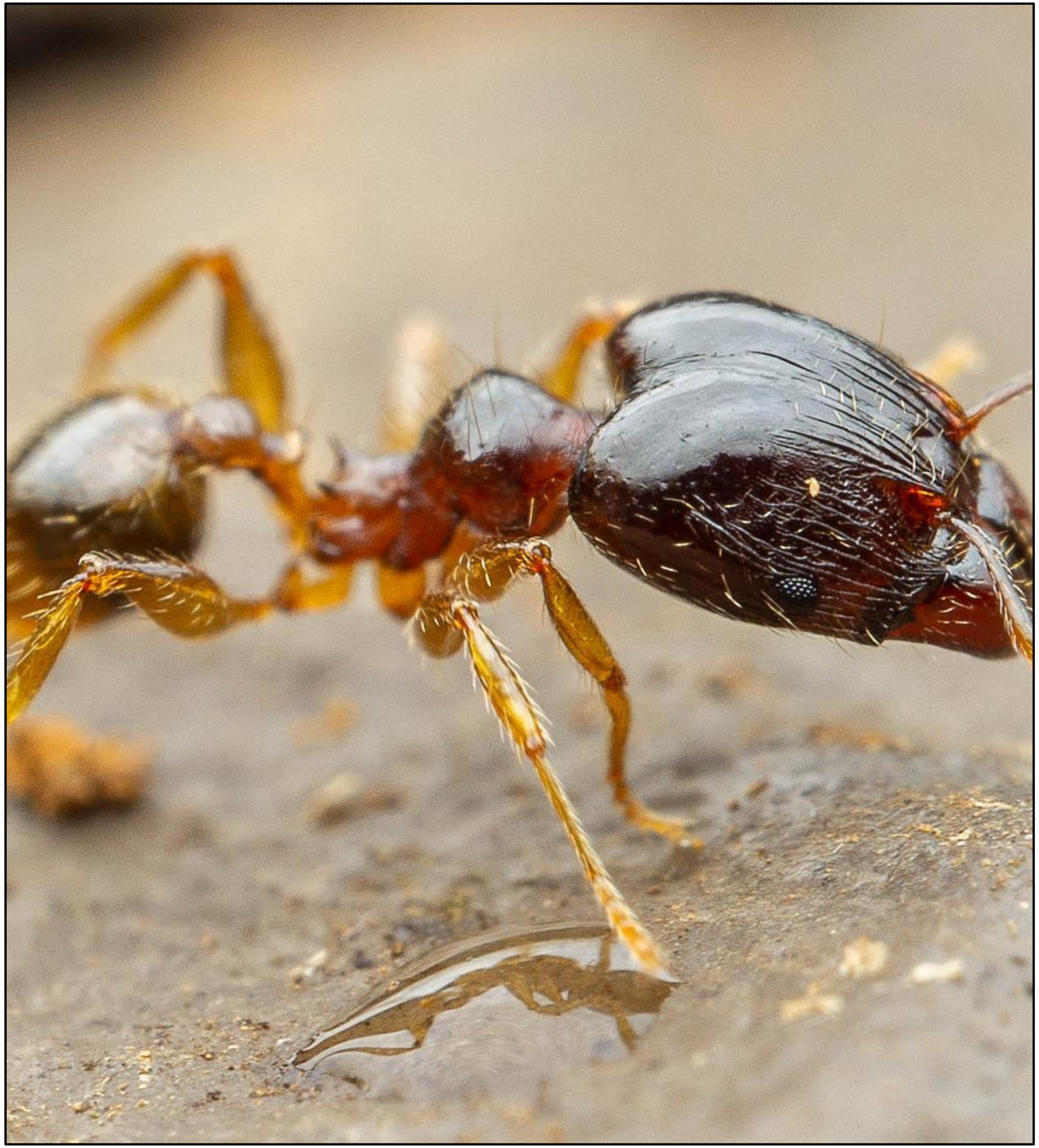


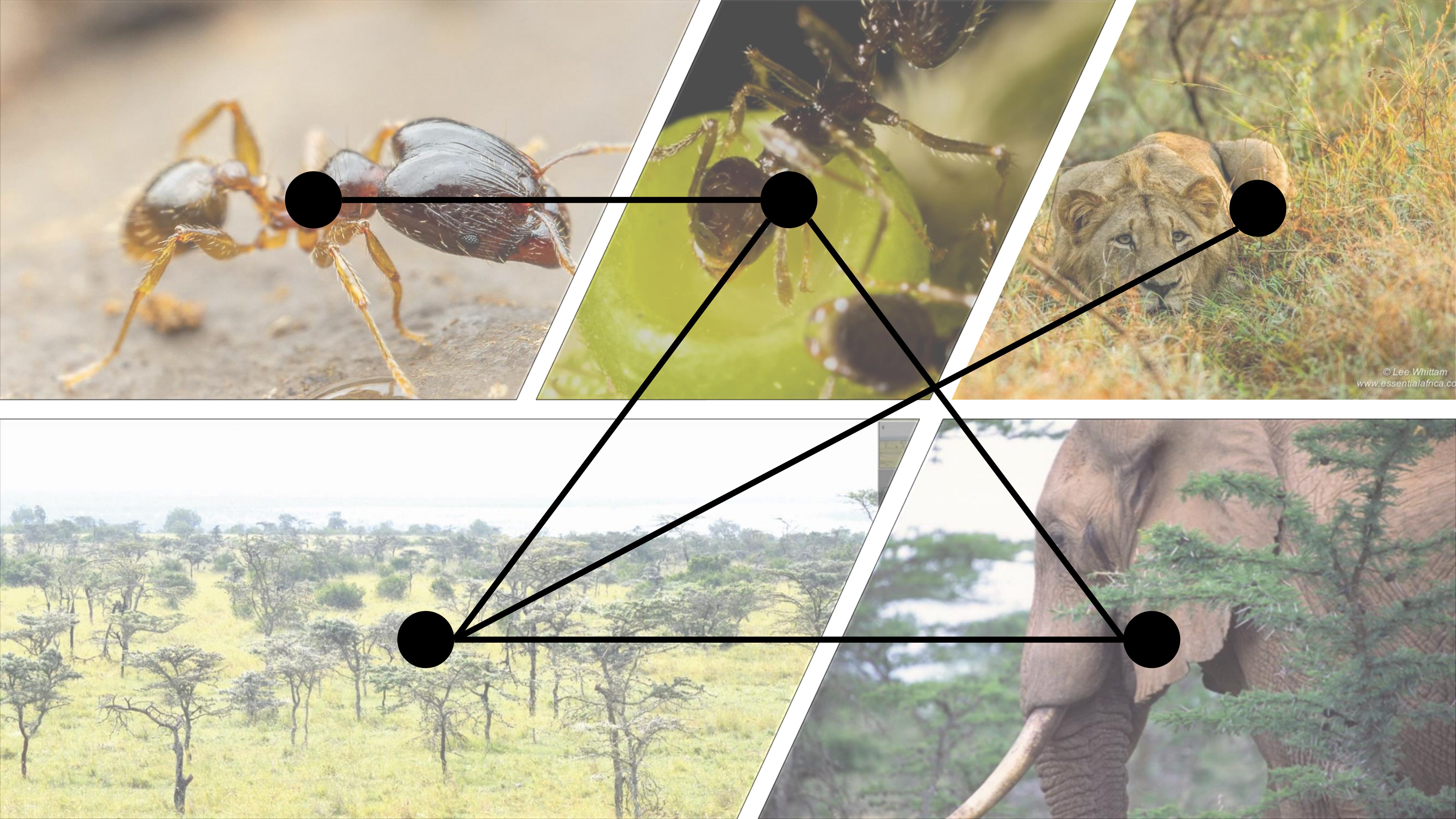
Species can affect each other through longer pathways!



How important are these longer pathways?



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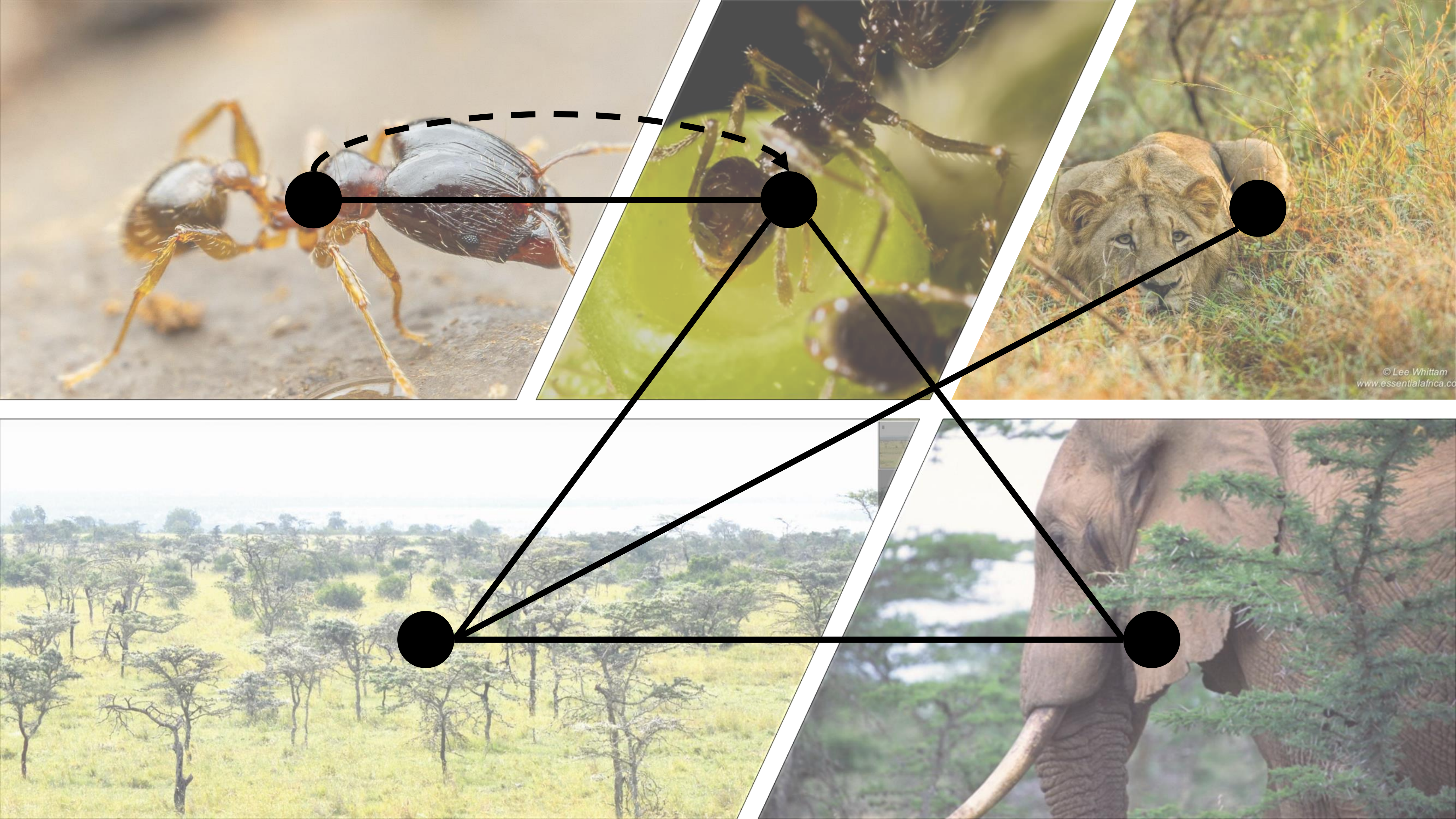
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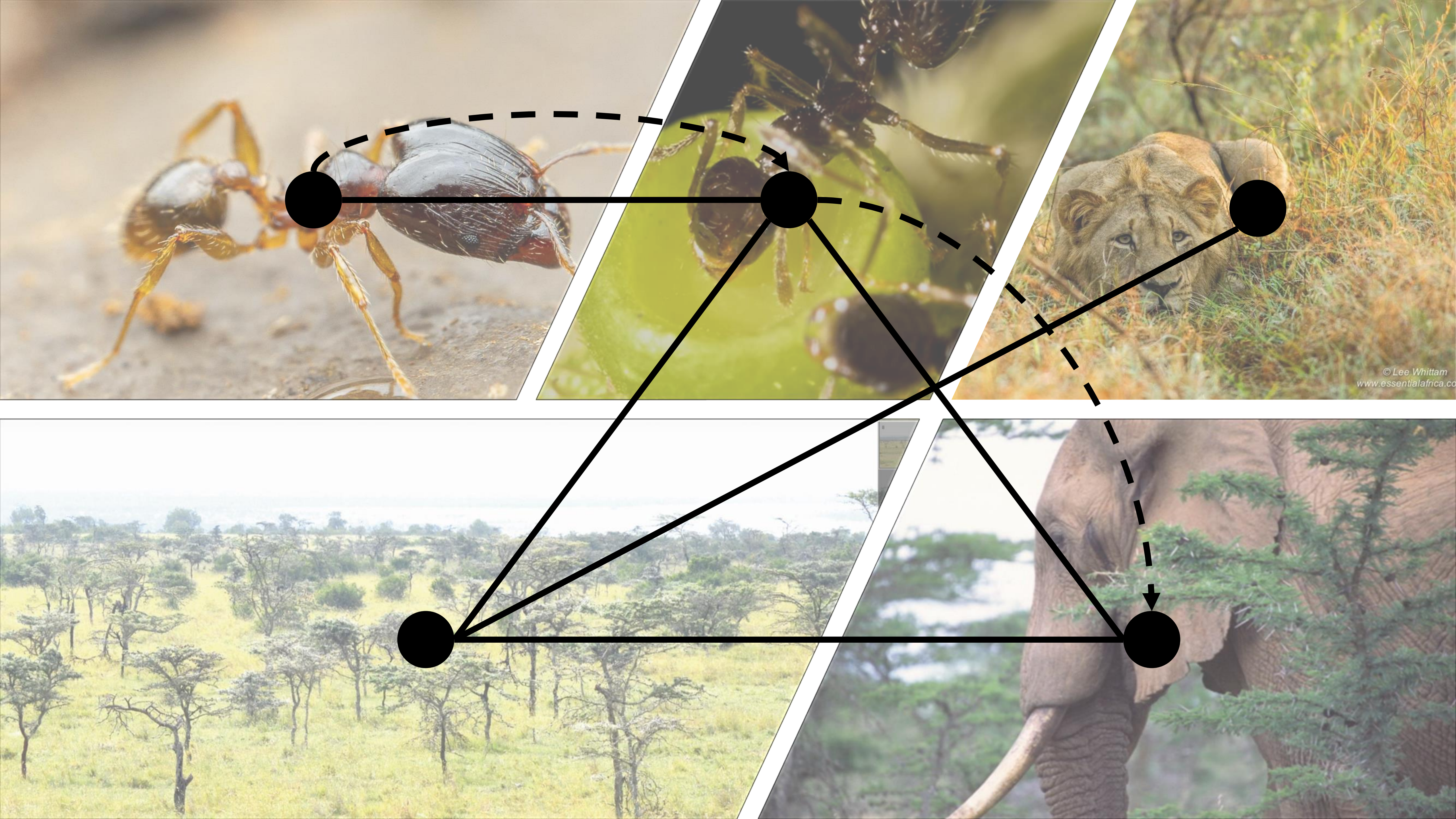
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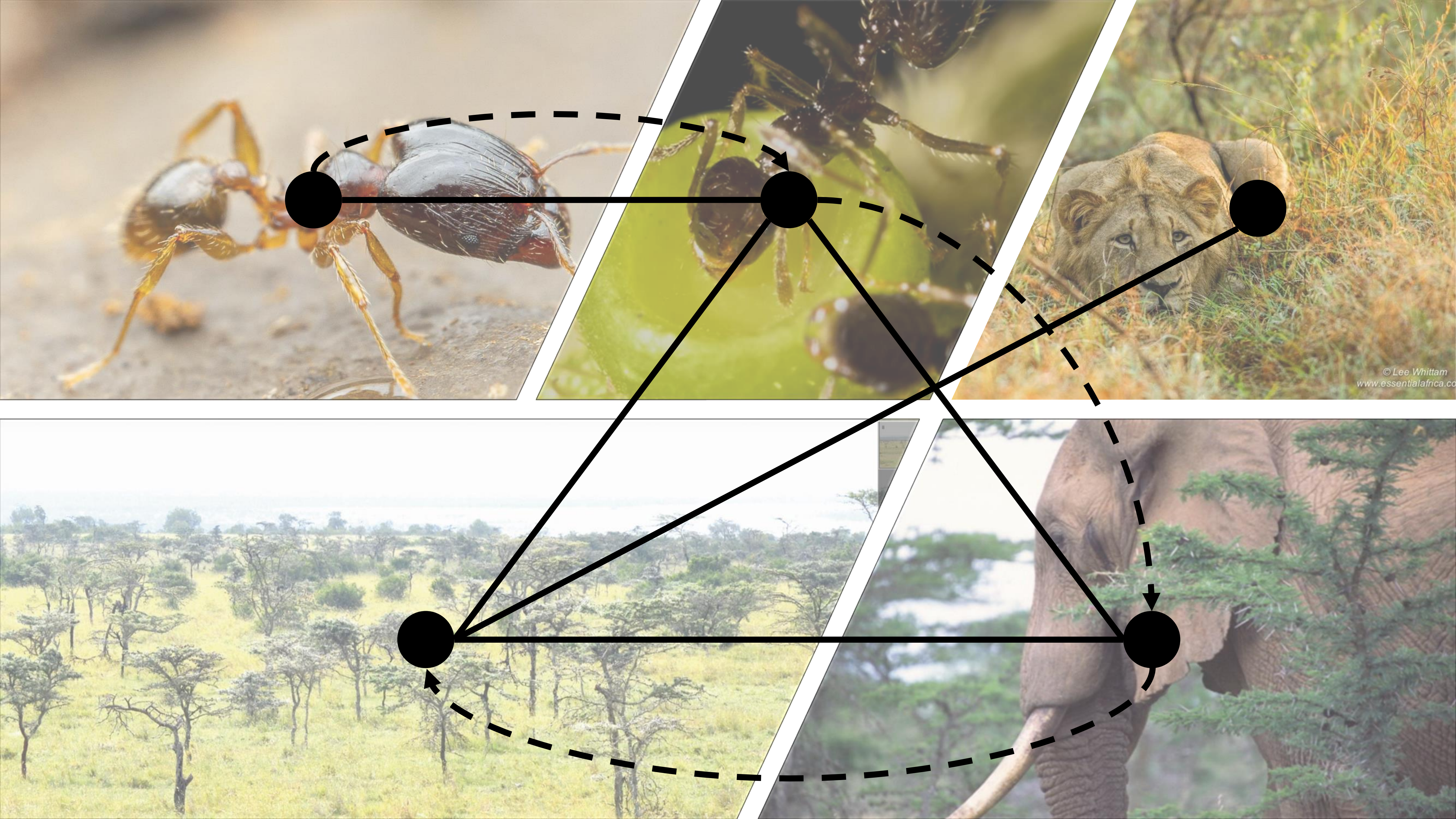
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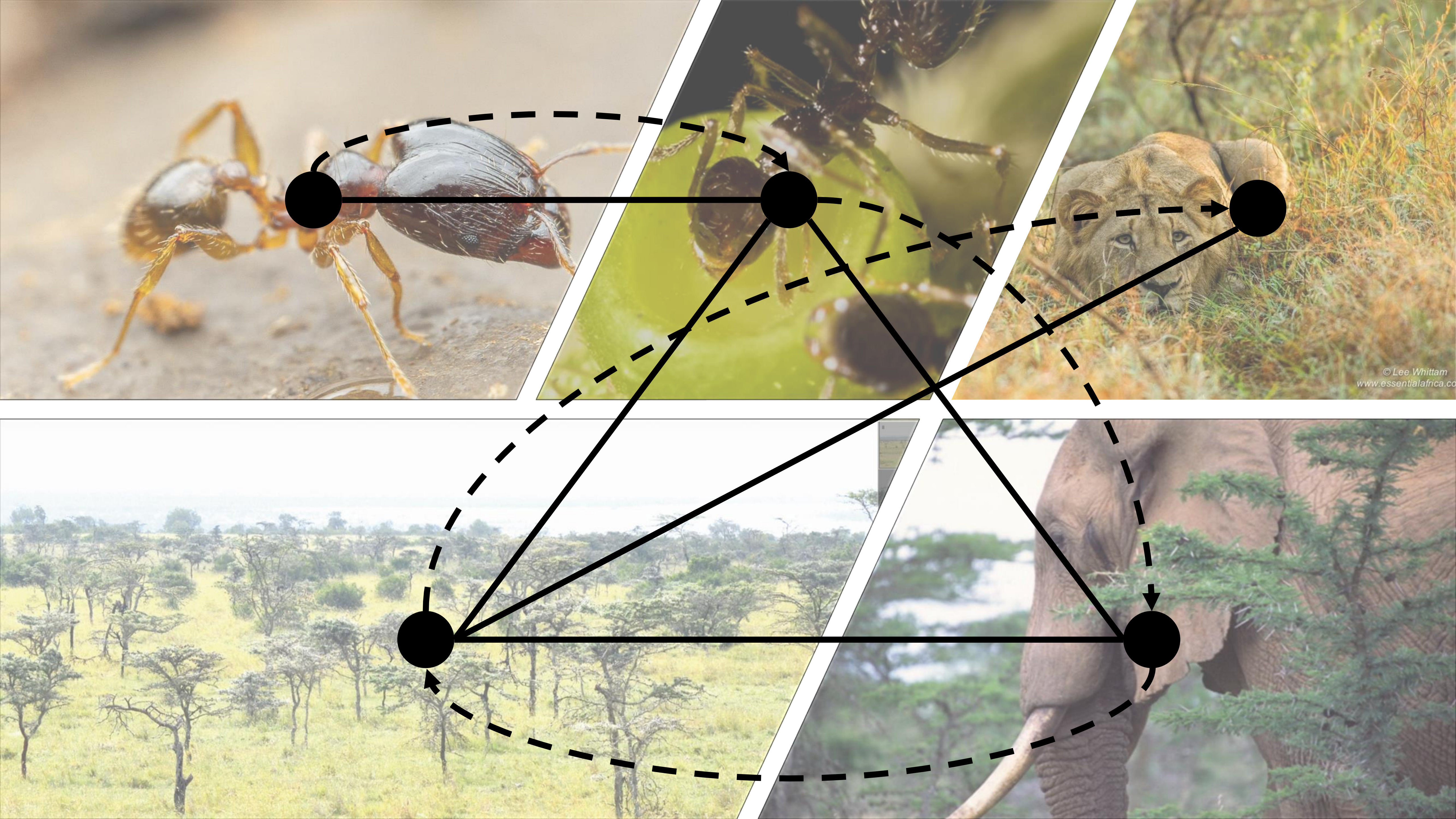
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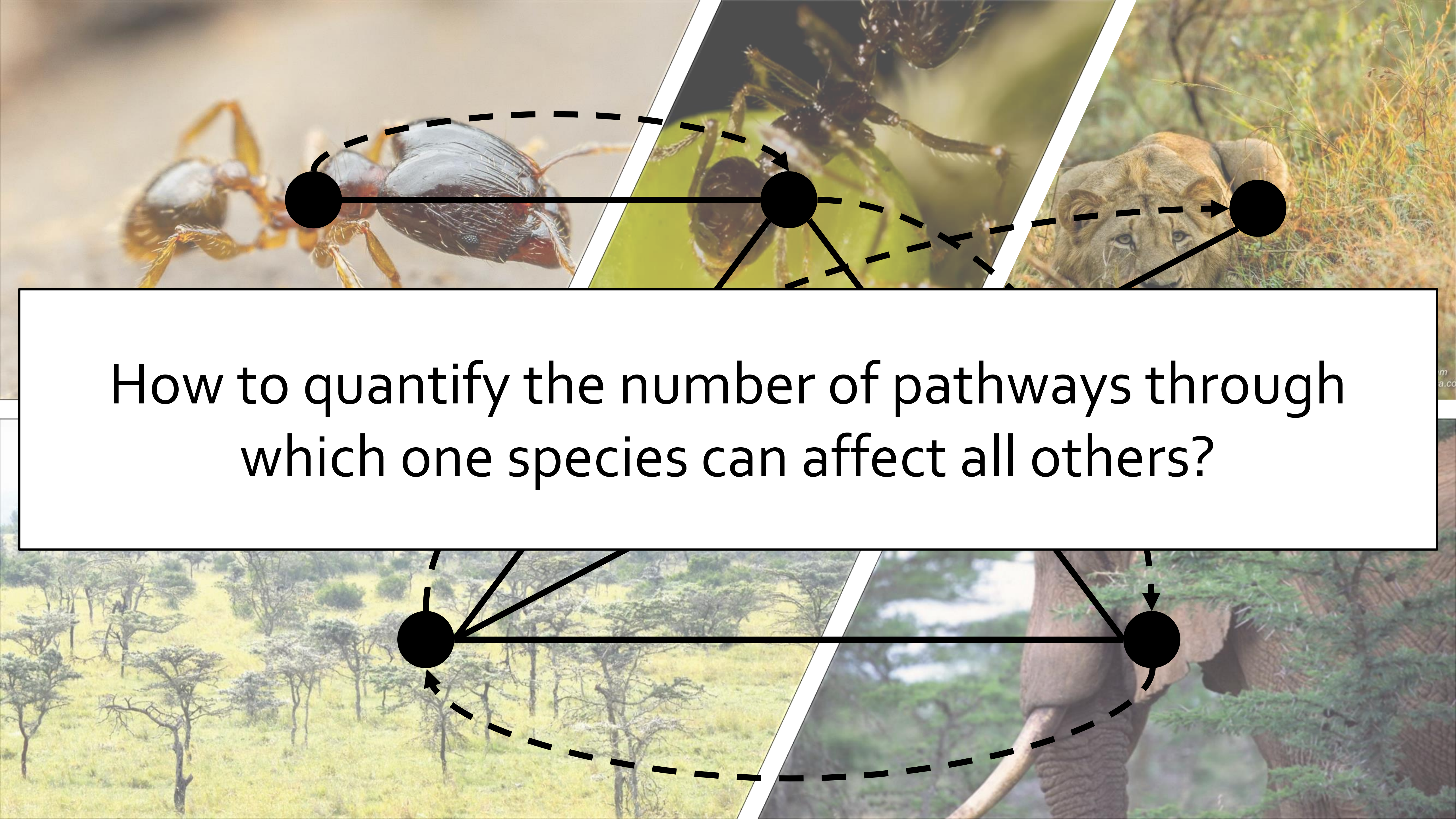
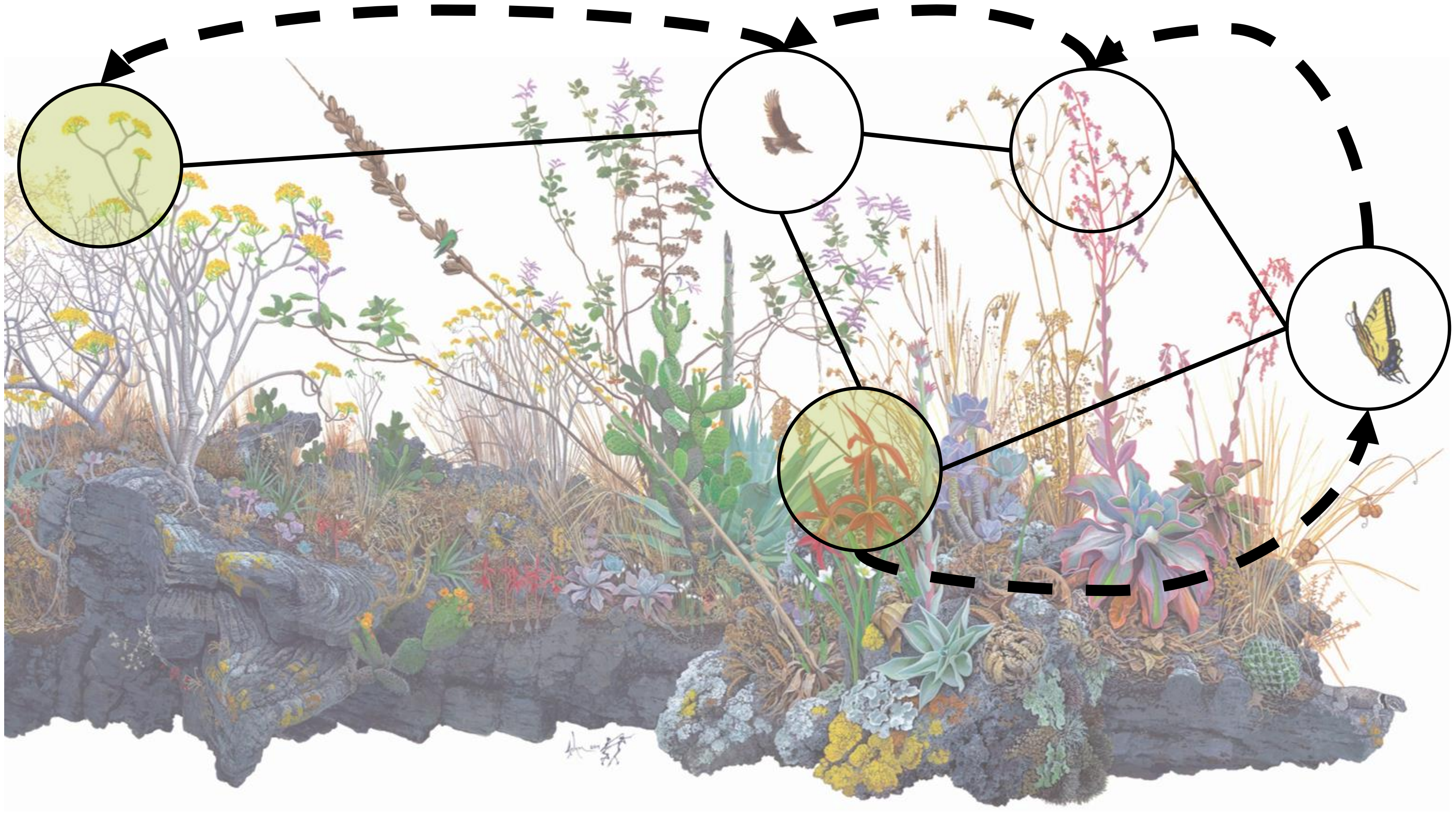


How to quantify the number of pathways through which one species can affect all others?



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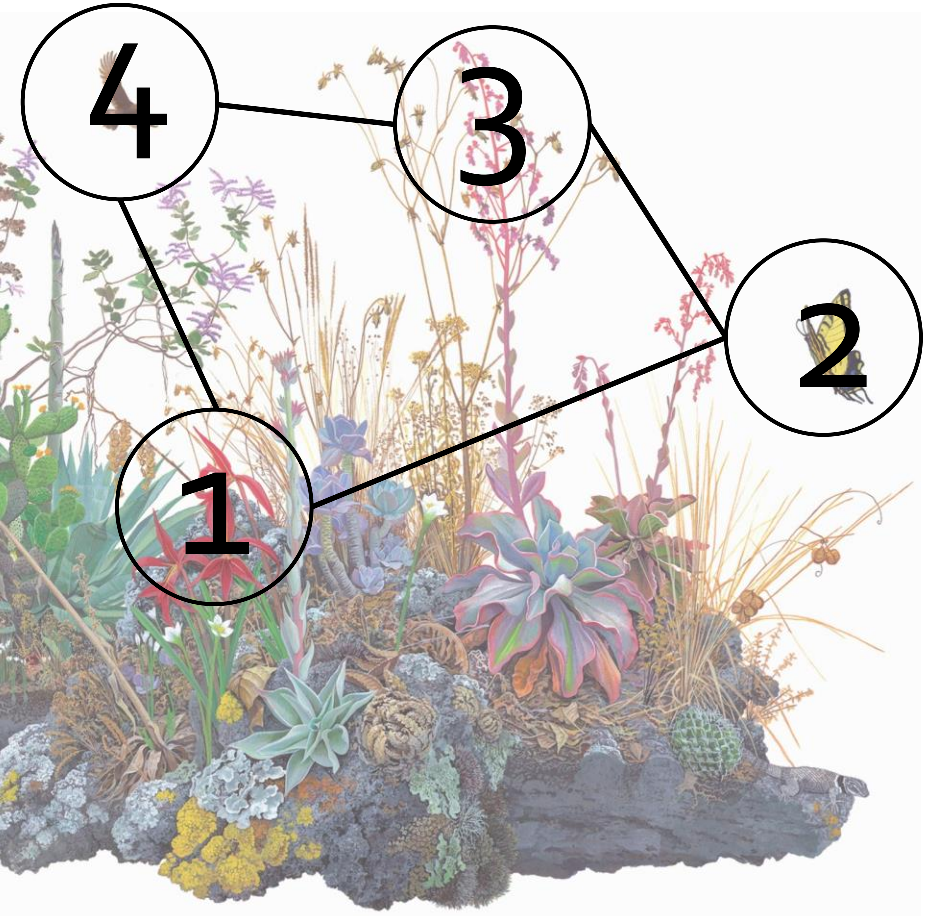
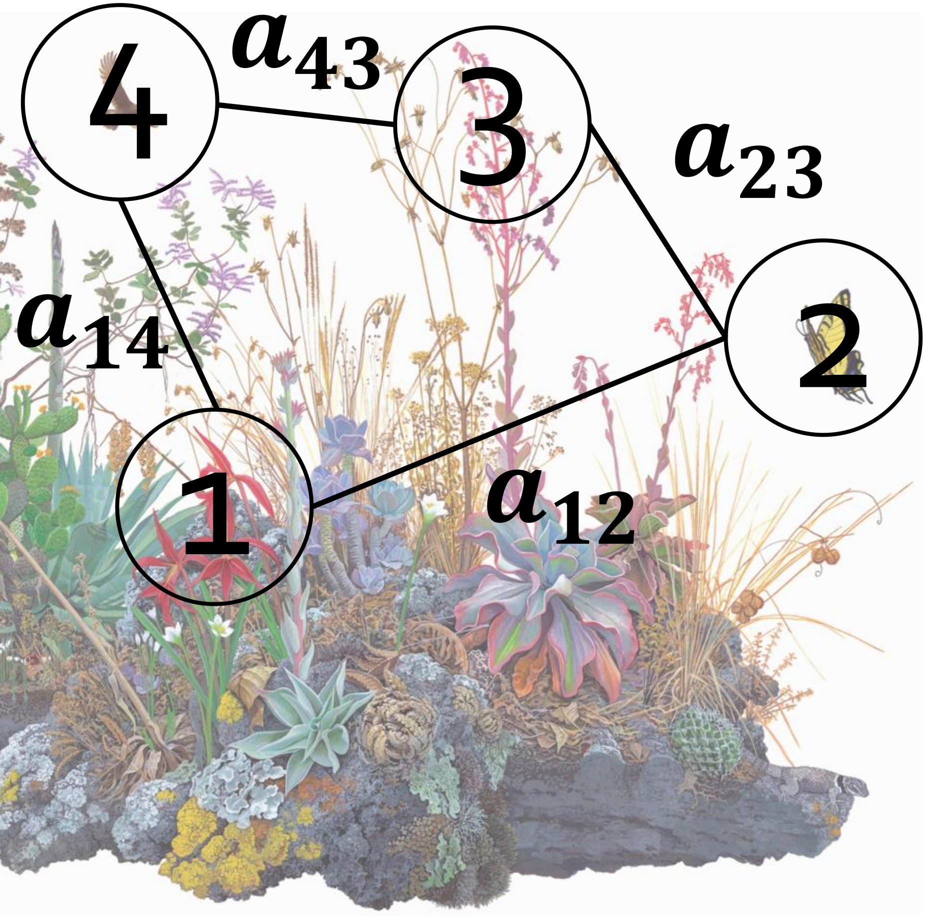
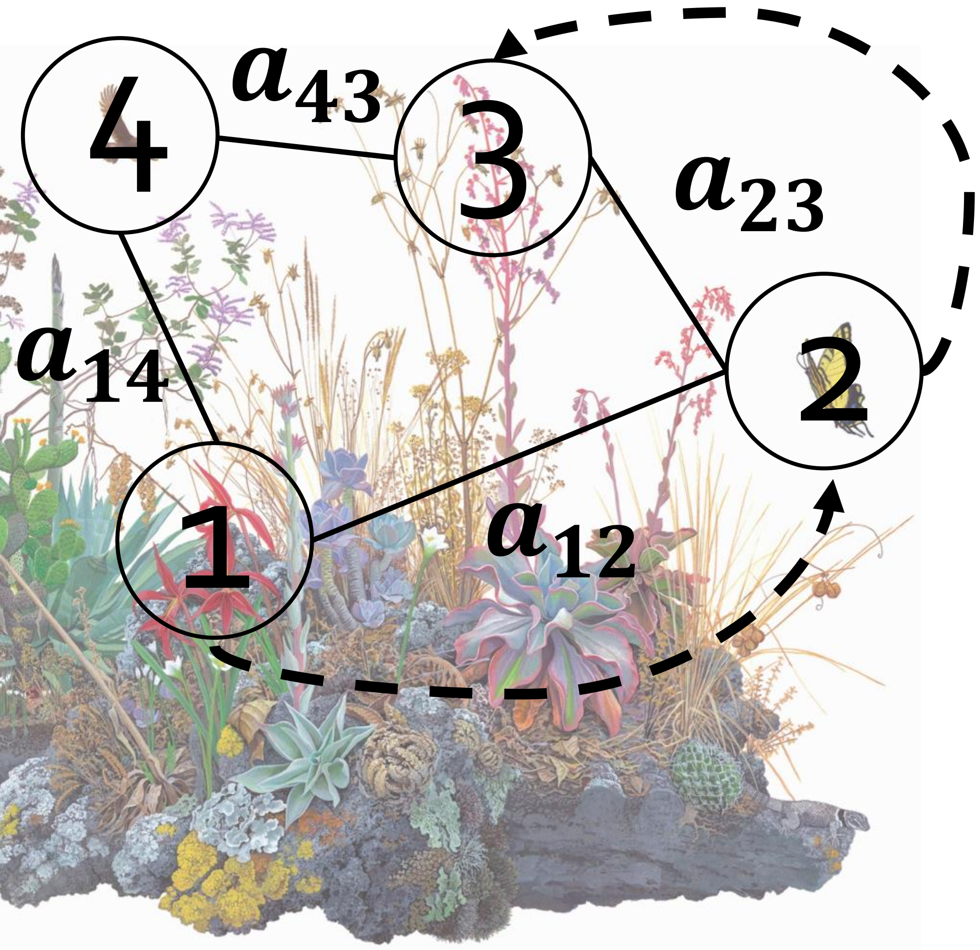
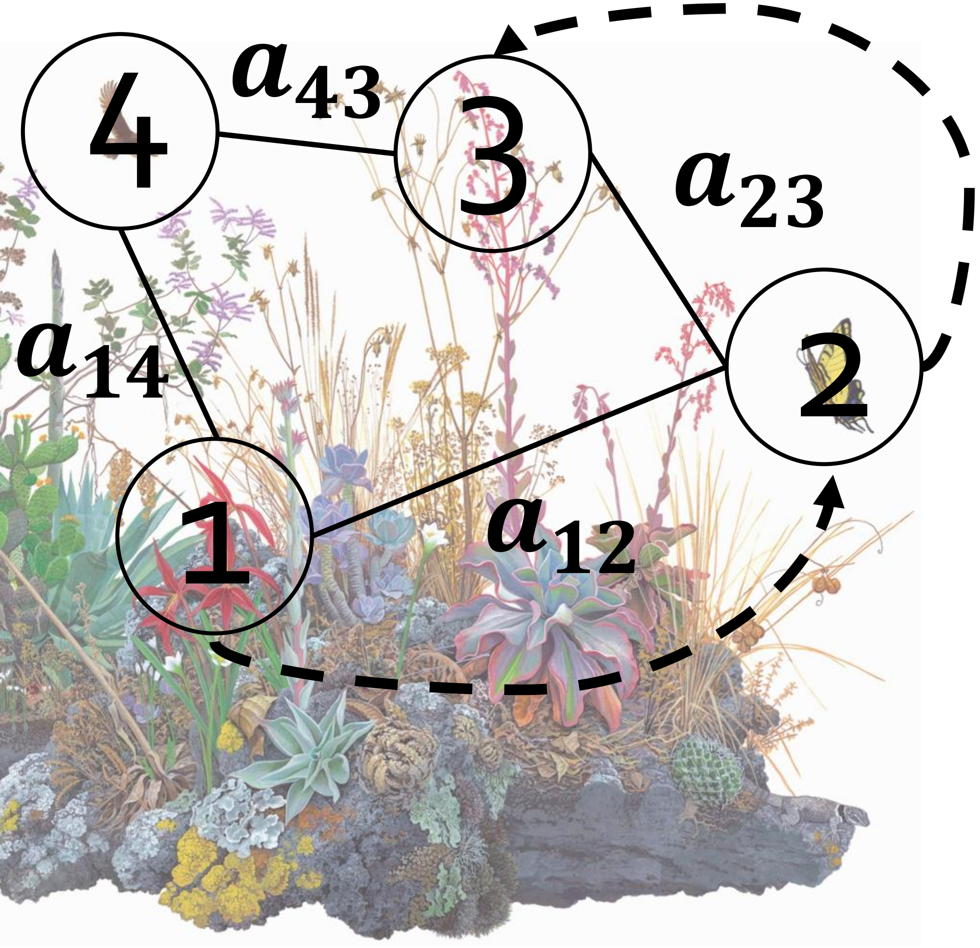

4
3
2
1



4
3
2
1
a_{43}
a_{23}
a_{14}
a_{12}

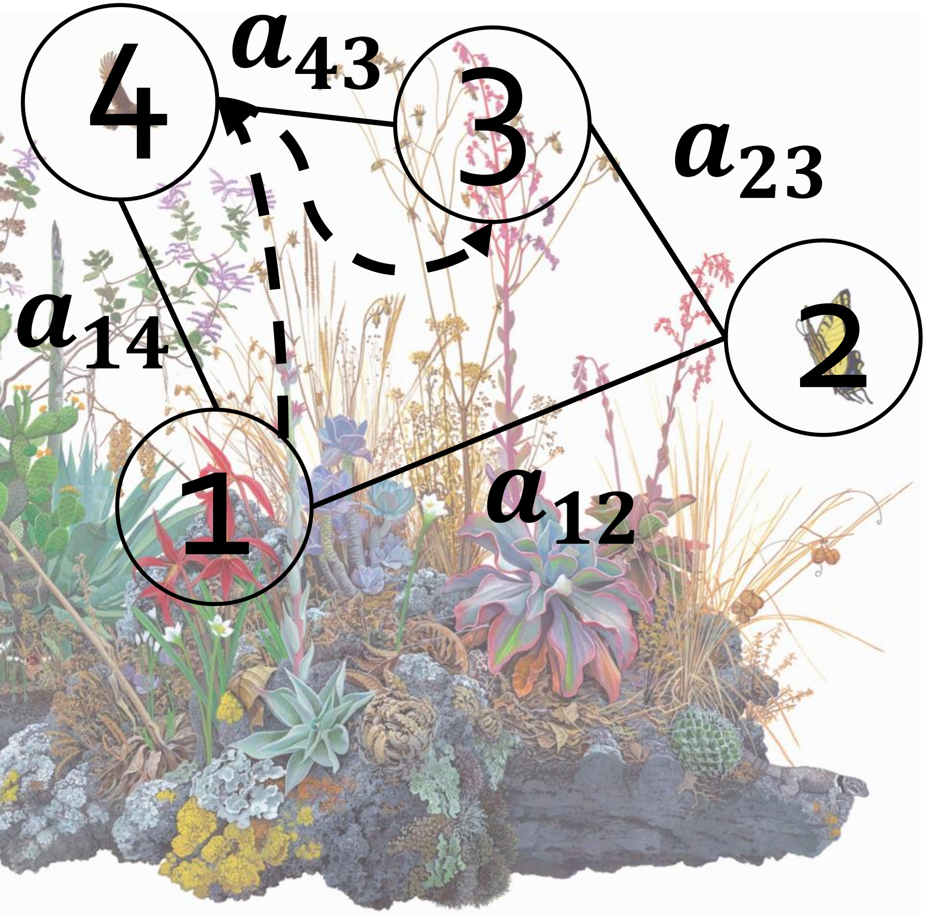


4
3
2
1
a_{43}
a_{23}
a_{14}
a_{12}



$$a_{12}a_{23}$$

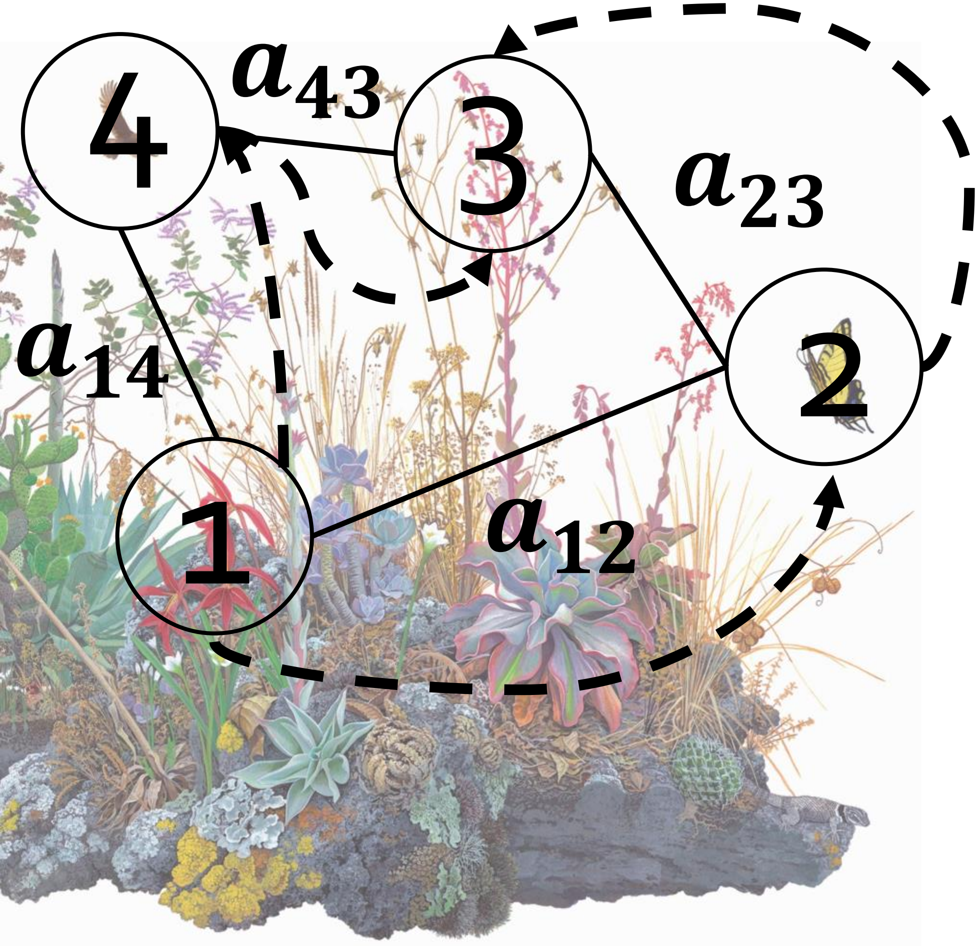
One pathway of length 2
Links species 1 and 3



$$a_{14}a_{43}$$

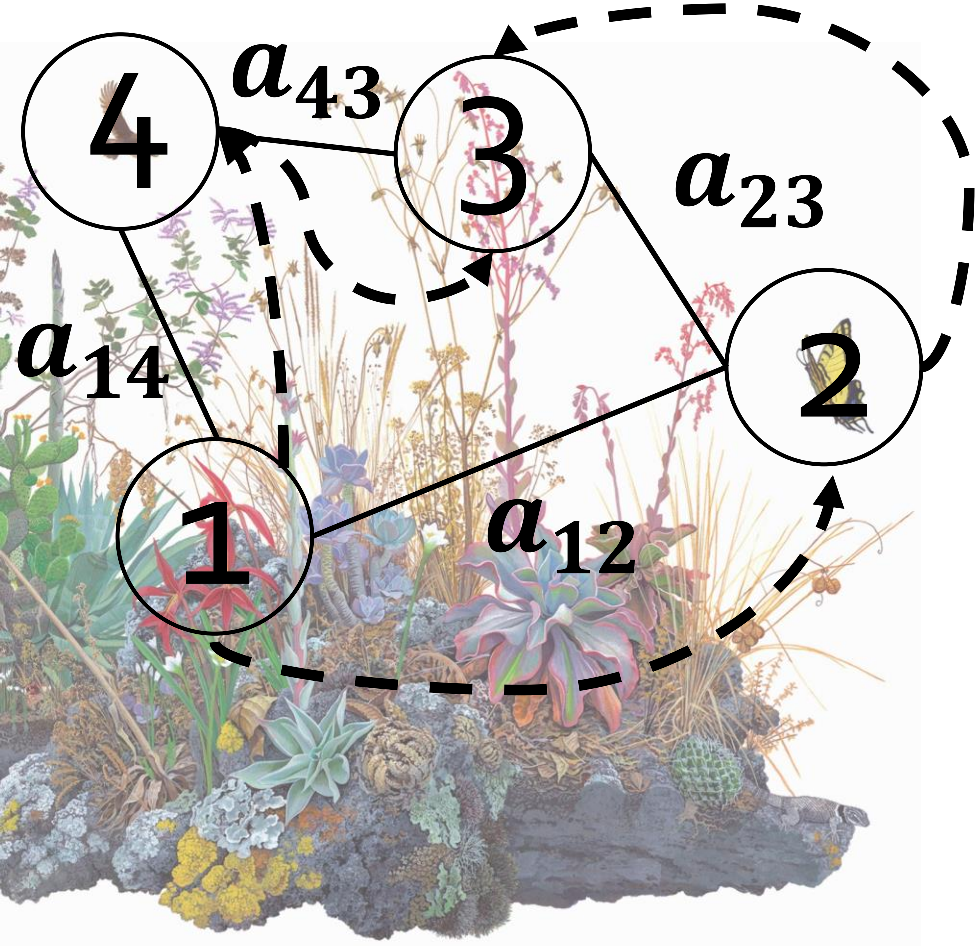
Another pathway of length 2

Links species 1 and 3



$$a_{12}a_{23} + a_{14}a_{43}$$

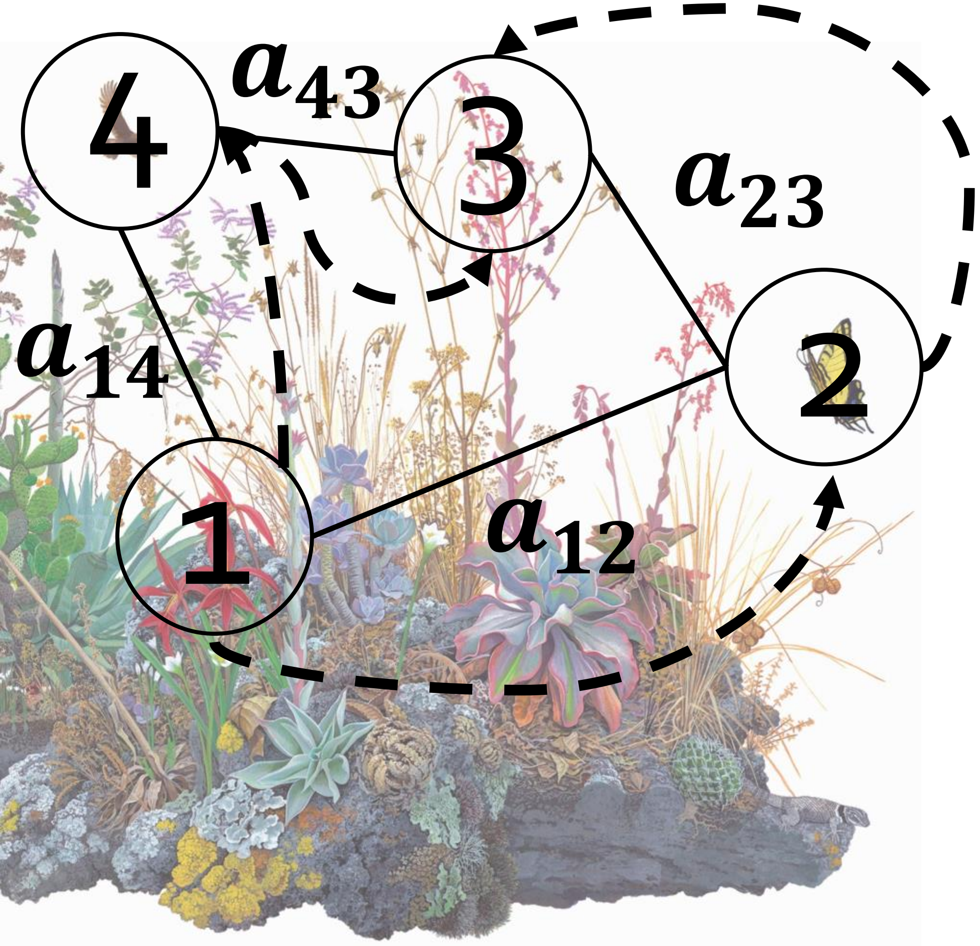
Sum of the pathways of length 2
Links species 1 and 3



$$a_{12}a_{23} + a_{14}a_{43}$$

Sum of the pathways of length 2
Links species 1 and 3

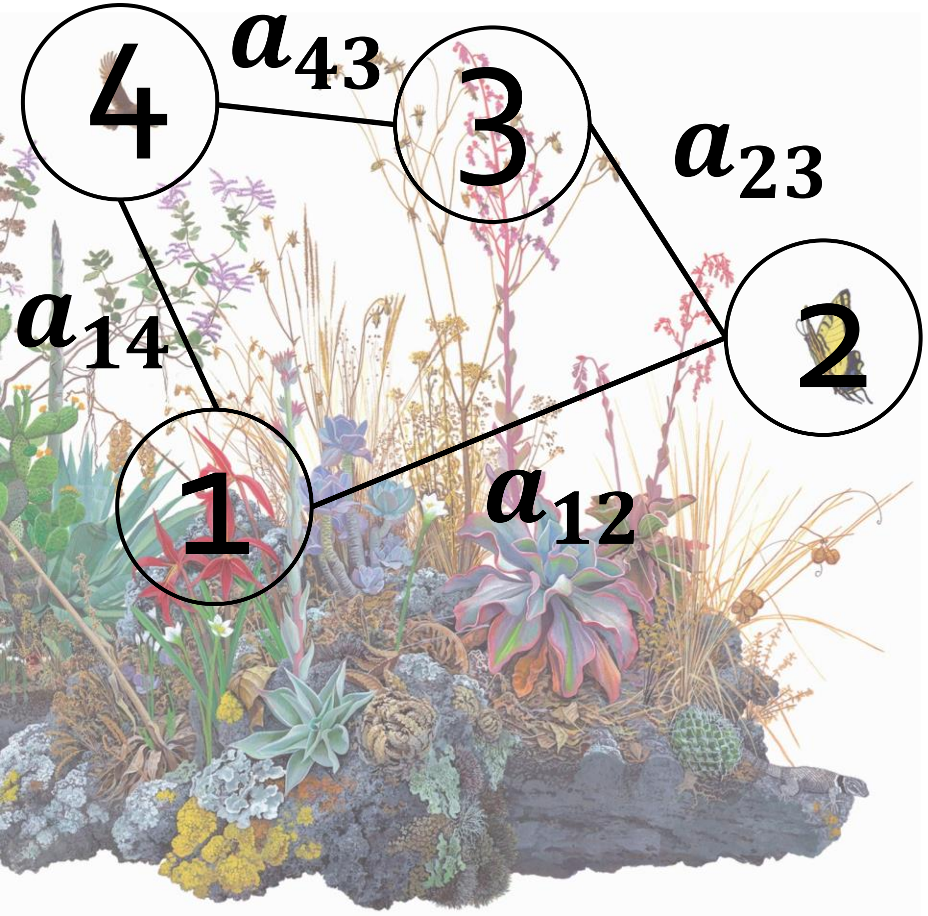
Do we need to do everything by hand?



$$a_{12}a_{23} + a_{14}a_{43}$$

Sum of the pathways of length 2
Links species 1 and 3

Thankfully not! Matrix multiplication does exactly that!



$$\mathbf{A} = \begin{pmatrix} 0 & a_{12} & \mathbf{0} & a_{14} \\ a_{21} & 0 & a_{23} & \mathbf{0} \\ \mathbf{0} & a_{32} & 0 & a_{34} \\ a_{41} & \mathbf{0} & a_{43} & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & \boldsymbol{a_{12}} & \mathbf{0} & \boldsymbol{a_{14}} \\ \boldsymbol{a_{21}} & 0 & \boldsymbol{a_{23}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{a_{32}} & 0 & \boldsymbol{a_{34}} \\ \boldsymbol{a_{41}} & \mathbf{0} & \boldsymbol{a_{43}} & 0 \end{pmatrix} \begin{pmatrix} 0 & \boldsymbol{a_{12}} & \mathbf{0} & \boldsymbol{a_{14}} \\ \boldsymbol{a_{21}} & 0 & \boldsymbol{a_{23}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{a_{32}} & 0 & \boldsymbol{a_{34}} \\ \boldsymbol{a_{41}} & \mathbf{0} & \boldsymbol{a_{43}} & 0 \end{pmatrix}$$

What happens when we multiply **A** by itself, or perform the operation $\mathbf{A}^2$?

$$\begin{pmatrix} 0 & a_{12} & \mathbf{0} & a_{14} \\ a_{21} & 0 & a_{23} & \mathbf{0} \\ \mathbf{0} & a_{32} & 0 & a_{34} \\ a_{41} & \mathbf{0} & a_{43} & 0 \end{pmatrix} \begin{pmatrix} 0 & a_{12} & \mathbf{0} & a_{14} \\ a_{21} & 0 & a_{23} & \mathbf{0} \\ \mathbf{0} & a_{32} & 0 & a_{34} \\ a_{41} & \mathbf{0} & a_{43} & 0 \end{pmatrix}$$

What happens when we multiply **A** by itself, or perform the operation **A**^{2} ?

$$
\begin{pmatrix}
0 & a_{12} & \mathbf{0} & a_{14} \\
\boldsymbol{a_{21}} & 0 & \boldsymbol{a_{23}} & \mathbf{0} \\
\mathbf{0} & \boldsymbol{a_{32}} & 0 & \boldsymbol{a_{34}} \\
\boldsymbol{a_{41}} & \mathbf{0} & \boldsymbol{a_{43}} & 0
\end{pmatrix}
\begin{pmatrix}
0 & \boldsymbol{a_{12}} & \mathbf{0} & \boldsymbol{a_{14}} \\
a_{21} & 0 & \boldsymbol{a_{23}} & \mathbf{0} \\
\mathbf{0} & \boldsymbol{a_{32}} & 0 & \boldsymbol{a_{34}} \\
a_{41} & \mathbf{0} & \boldsymbol{a_{43}} & 0
\end{pmatrix}
$$

$$
0.0 + a_{12}\, a_{21} + 0.0 + a_{14} a_{41} = \boldsymbol{a_{12}\, a_{21} + a_{14} a_{41}}
$$

$$\begin{pmatrix} 0 & a_{12} & 0 & a_{14} \\ a_{21} & 0 & a_{23} & 0 \\ 0 & a_{32} & 0 & a_{34} \\ a_{41} & 0 & a_{43} & 0 \end{pmatrix} \begin{pmatrix} 0 & a_{12} & 0 & a_{14} \\ a_{21} & 0 & a_{23} & 0 \\ 0 & a_{32} & 0 & a_{34} \\ a_{41} & 0 & a_{43} & 0 \end{pmatrix}$$

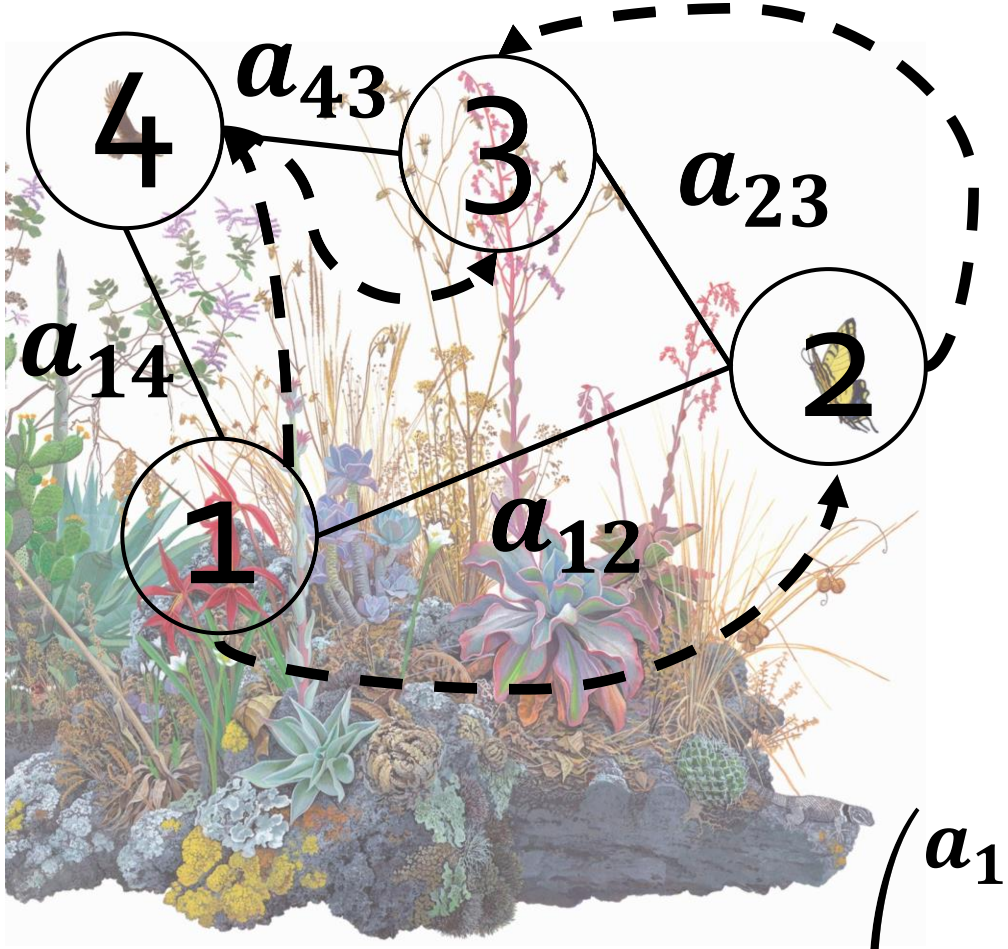
$$0.\, a_{12} + a_{12}.\, 0 + 0.\, a_{32} + a_{14}.\, 0$$

$$\begin{pmatrix} 0 & a_{12} & 0 & a_{14} \\ a_{21} & 0 & a_{23} & 0 \\ 0 & a_{32} & 0 & a_{34} \\ a_{41} & 0 & a_{43} & 0 \end{pmatrix} \begin{pmatrix} 0 & a_{12} & 0 & a_{14} \\ a_{21} & 0 & a_{23} & 0 \\ 0 & a_{32} & 0 & a_{34} \\ a_{41} & 0 & a_{43} & 0 \end{pmatrix}$$

$$0.0 + a_{12}.\,a_{23} + 0.0 + a_{14}.\,a_{43} = \boldsymbol{a_{12}.\,a_{23} + a_{14}.\,a_{43}}$$

$$\begin{pmatrix} 0 & \boldsymbol{a_{12}} & \mathbf{0} & \boldsymbol{a_{14}} \\ \boldsymbol{a_{21}} & 0 & \boldsymbol{a_{23}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{a_{32}} & 0 & \boldsymbol{a_{34}} \\ \boldsymbol{a_{41}} & \mathbf{0} & \boldsymbol{a_{43}} & 0 \end{pmatrix} \begin{pmatrix} 0 & \boldsymbol{a_{12}} & \mathbf{0} & \boldsymbol{a_{14}} \\ \boldsymbol{a_{21}} & 0 & \boldsymbol{a_{23}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{a_{32}} & 0 & \boldsymbol{a_{34}} \\ \boldsymbol{a_{41}} & \mathbf{0} & \boldsymbol{a_{43}} & 0 \end{pmatrix} =$$

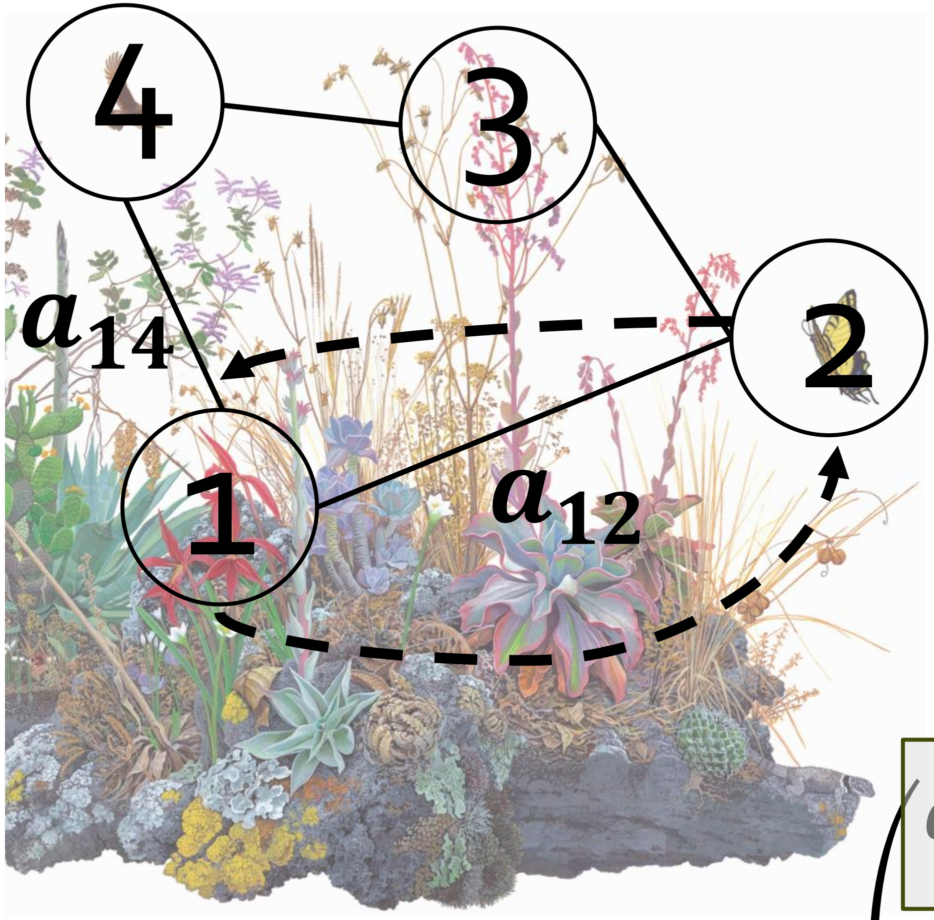
$$\begin{pmatrix} \boldsymbol{a_{12}a_{21} + a_{14}a_{41}} & \mathbf{0} & \boldsymbol{a_{12}a_{23} + a_{14}a_{43}} & \mathbf{0} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}$$



$$a_{12}a_{23} + a_{14}a_{43}$$

Sum of the pathways of length 2
Linking species 1 and 3

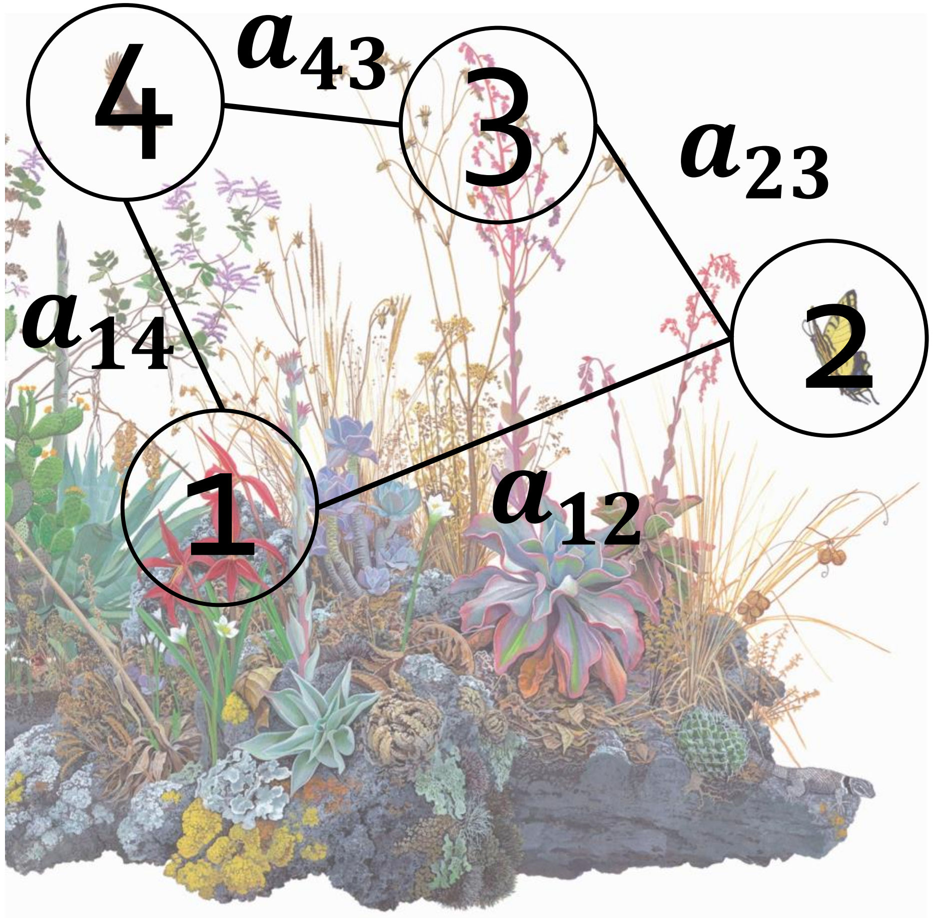
$$\begin{pmatrix} a_{12}a_{21} + a_{14}a_{41} & 0 & a_{12}a_{23} + a_{14}a_{43} & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$



$$a_{12}a_{21} + a_{14}a_{41}$$

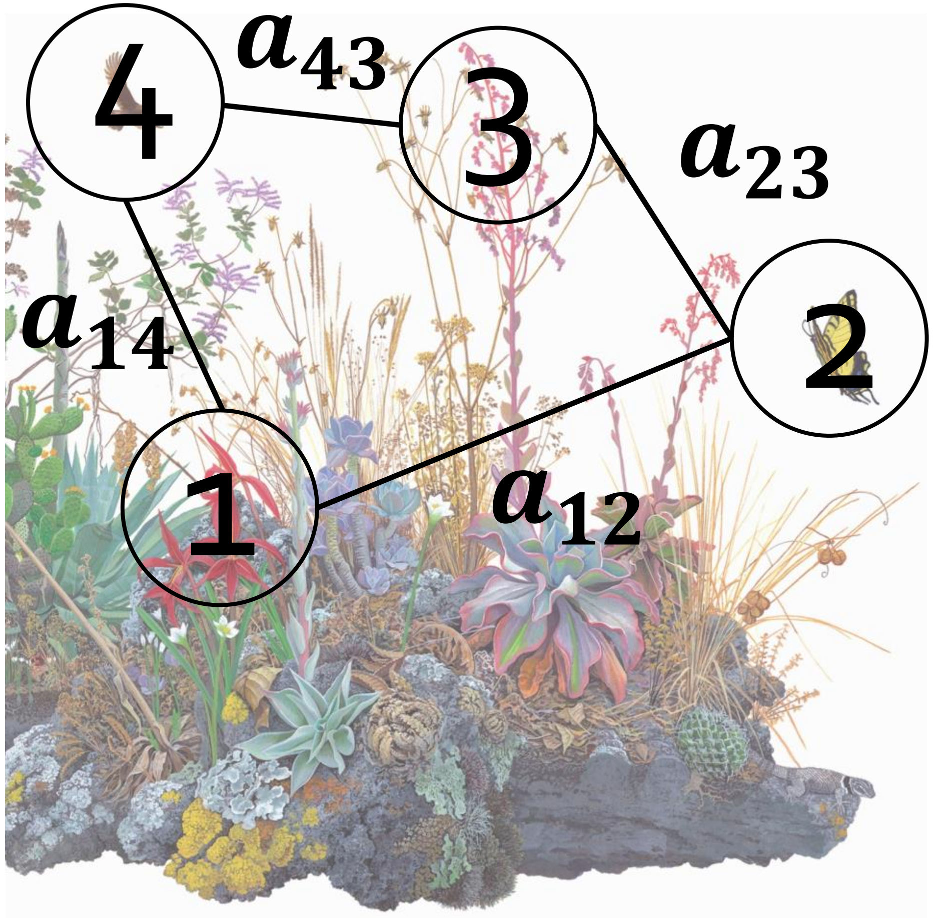
Sum of the pathways of length 2
Linking species 1 to itself!

$$\begin{pmatrix} a_{12}a_{21} + a_{14}a_{41} & 0 & a_{12}a_{23} + a_{14}a_{43} & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$



A^2

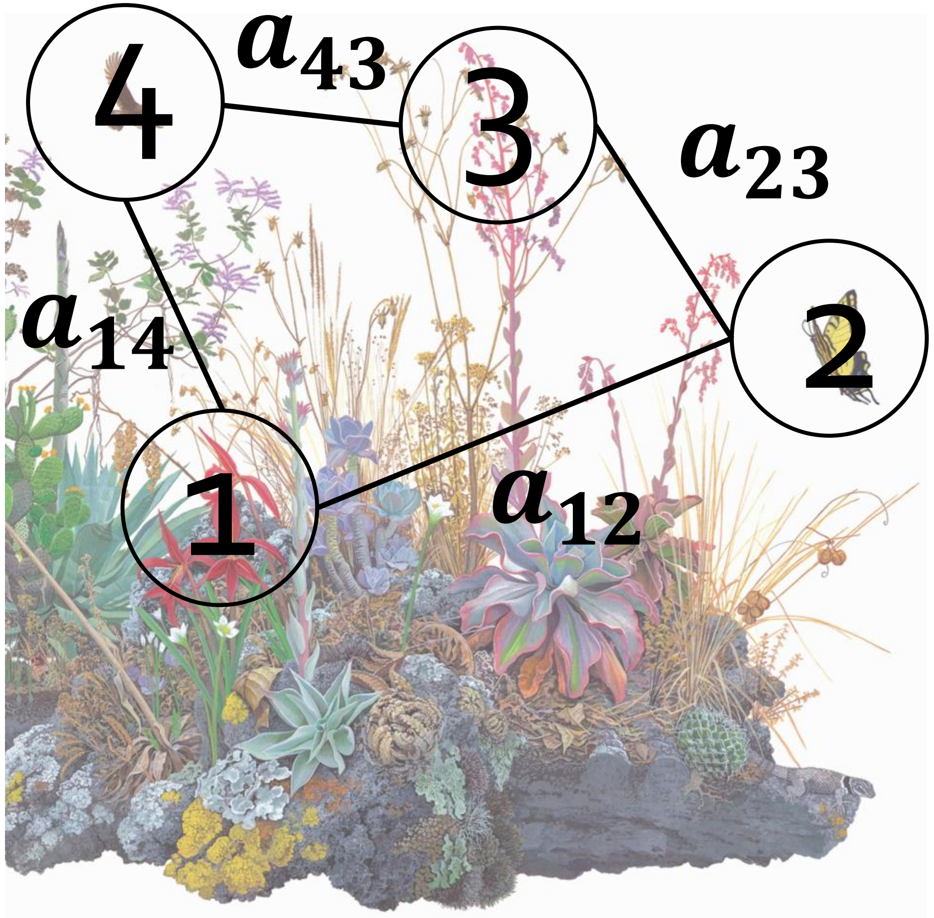
Sum of the pathways of length 2
Linking species in the network



$$A + A^2$$

Sum of the pathways of length 1 and 2

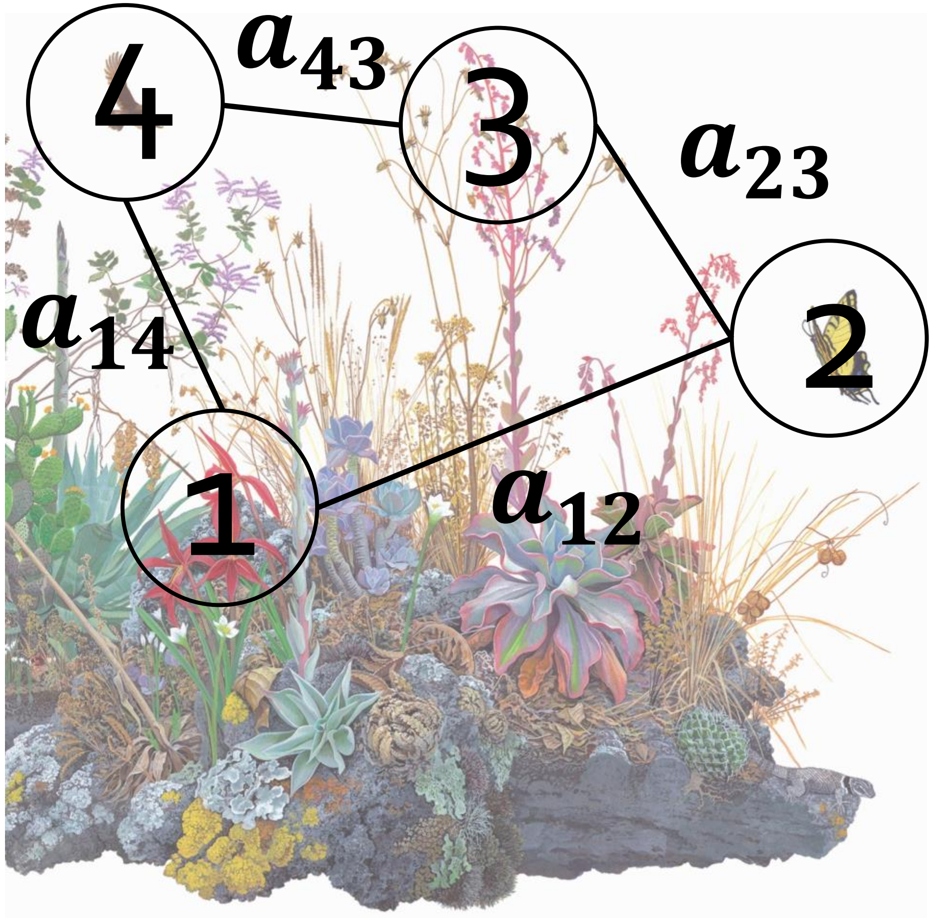
Linking species in the network



$$A + A^2 + A^3$$

Sum of the pathways of length 1, 2 and 3

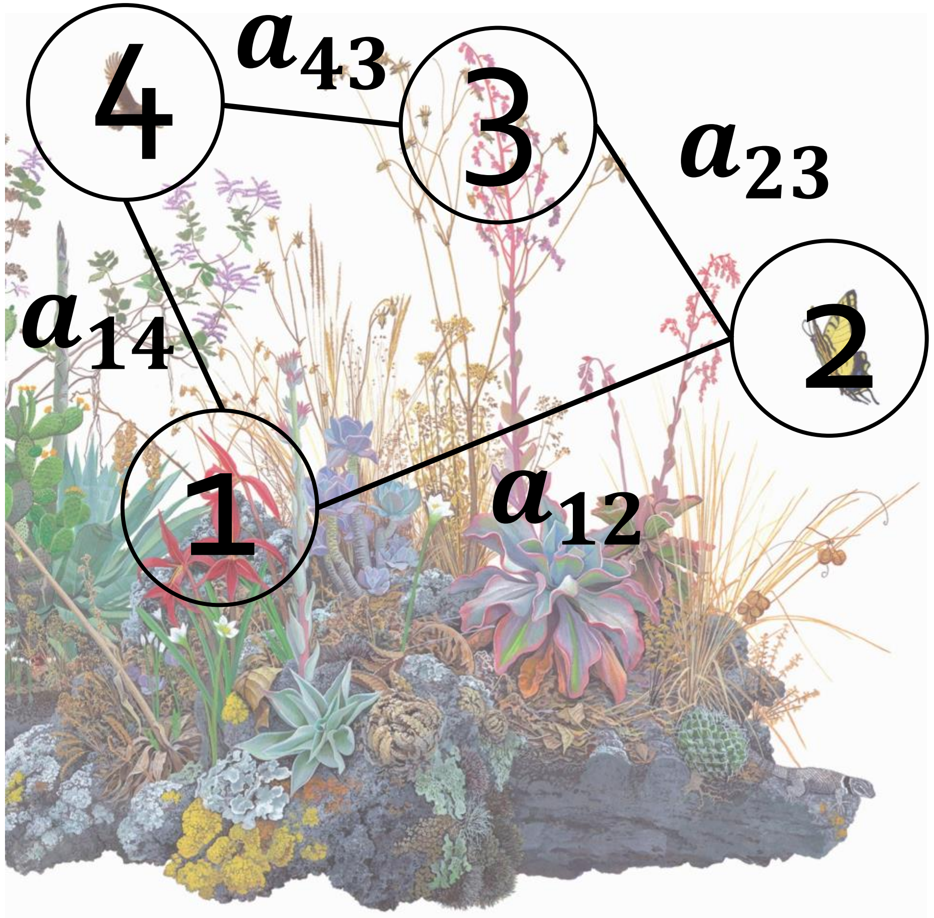
Linking species in the network



$$A + A^2 + A^3 \dots$$

Sum of the pathways of all possible lenghts?

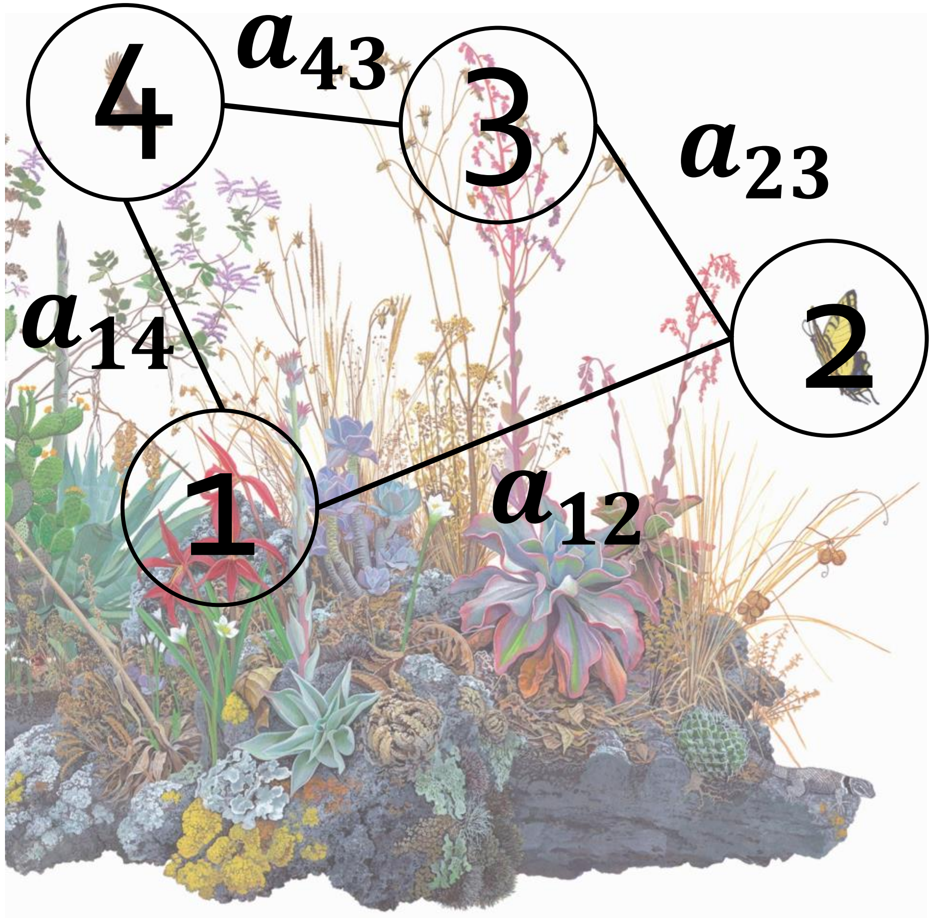
Linking species in the network



$$\sum_{k=0}^{\infty} A^k = (I - A)^{-1}$$

Sum of the pathways of all possible lenghts?

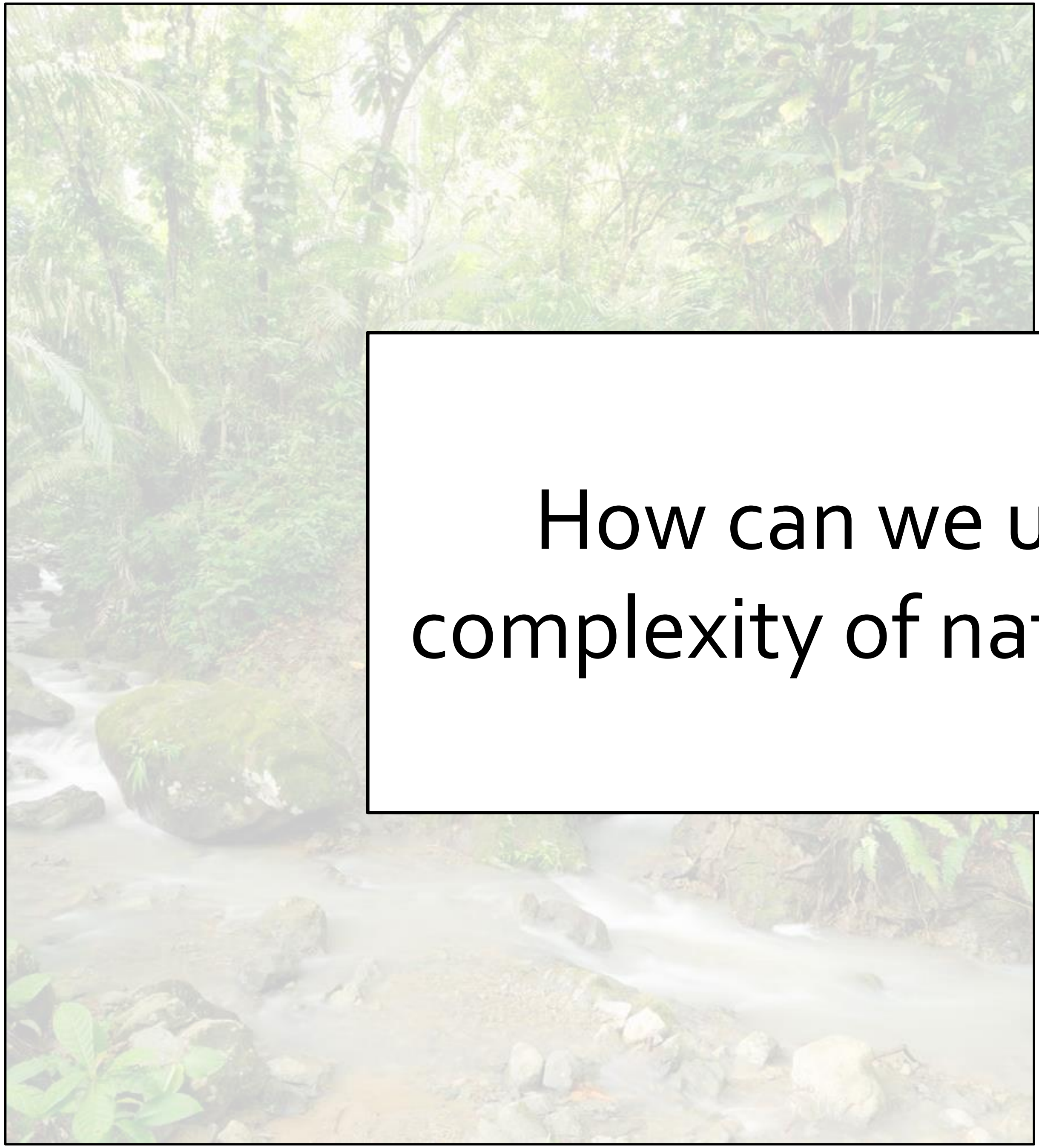
Sum of the rows of matrix $(I - A)^{-1}$

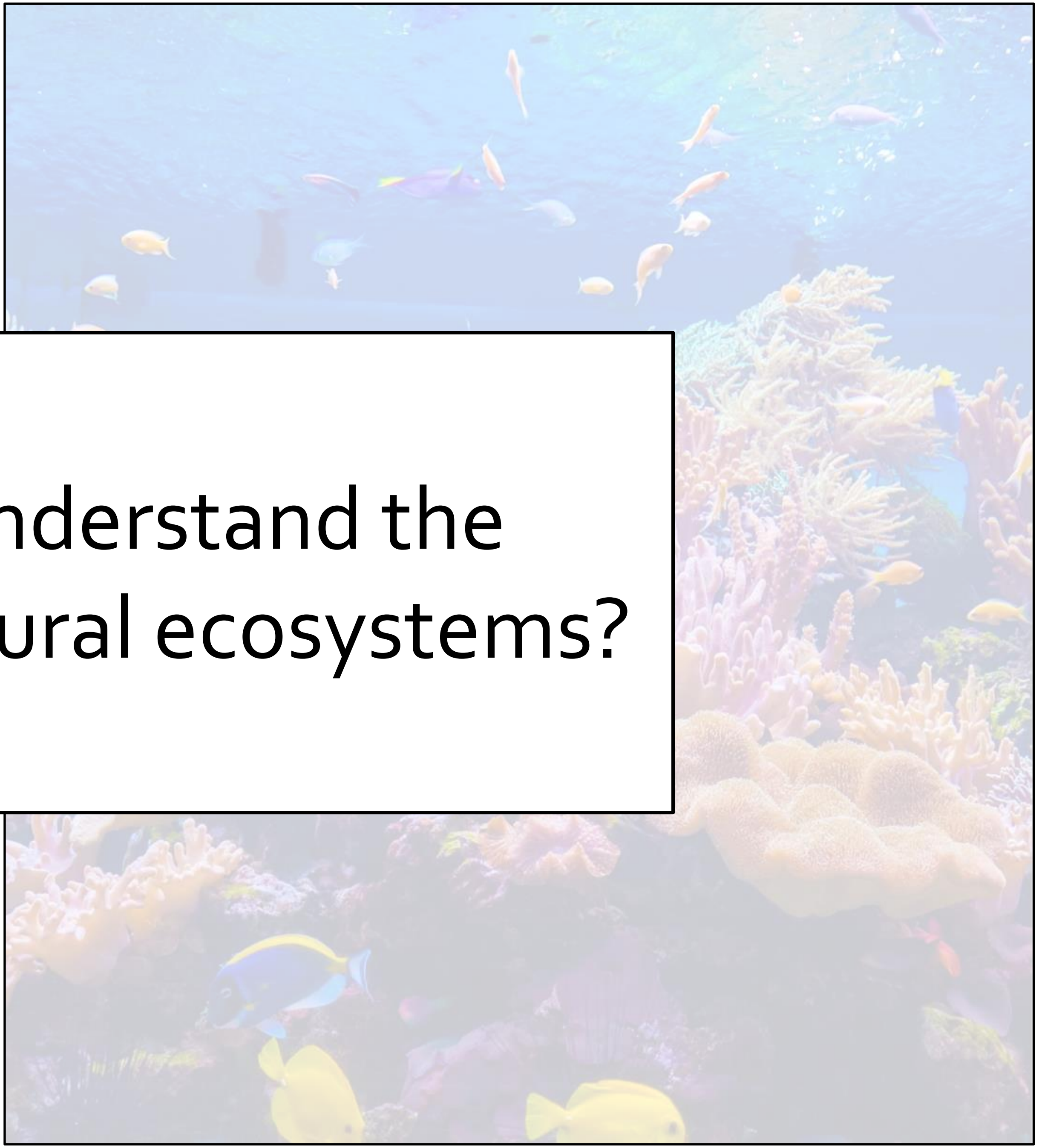


$$\sum_{k=0}^{\infty} \boldsymbol{A}^{k} = (\boldsymbol{I} - \boldsymbol{A})^{-1}$$

Sum of the pathways of all possible lenghts?

Katz centrality





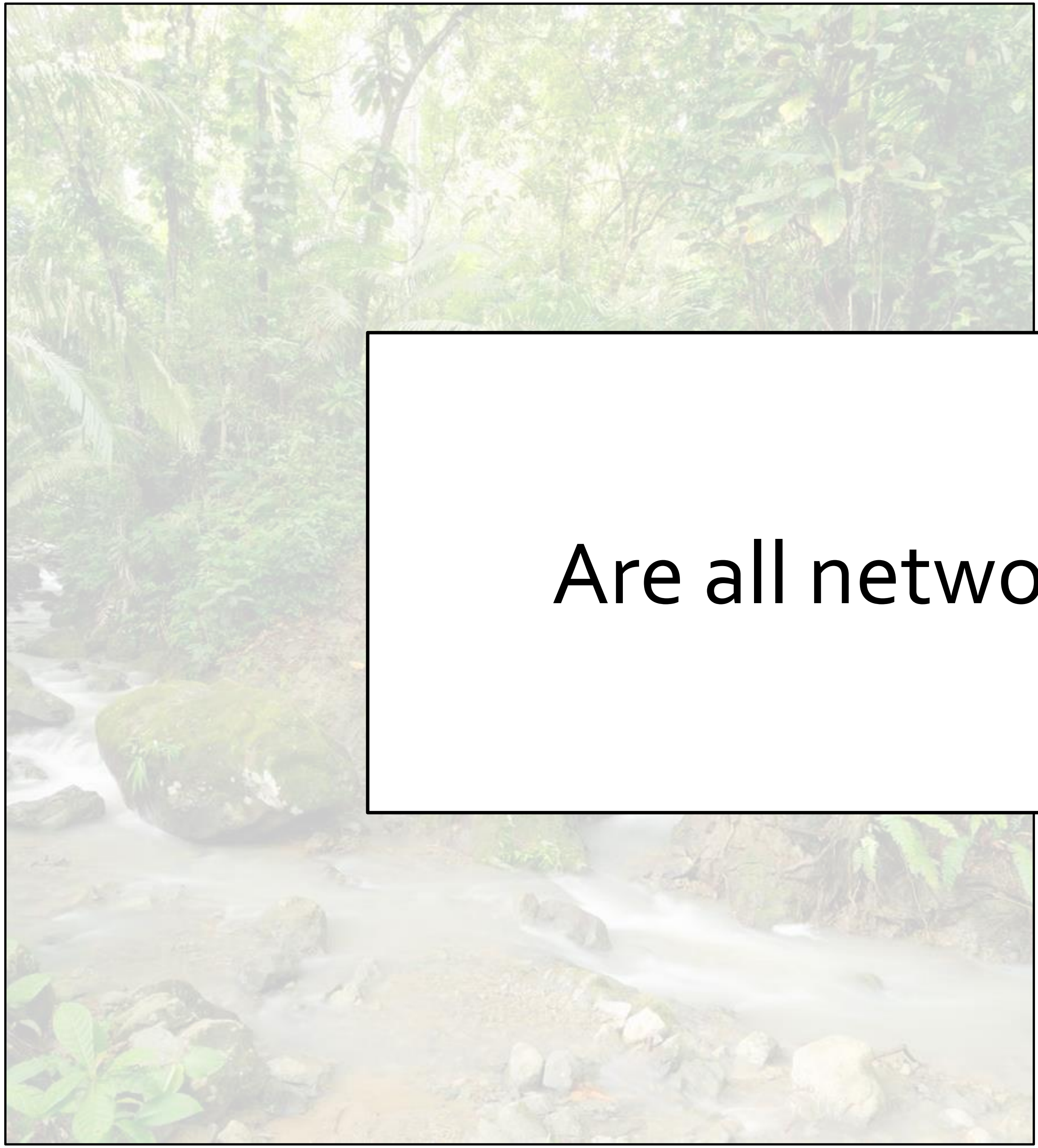
How can we understand the complexity of natural ecosystems?

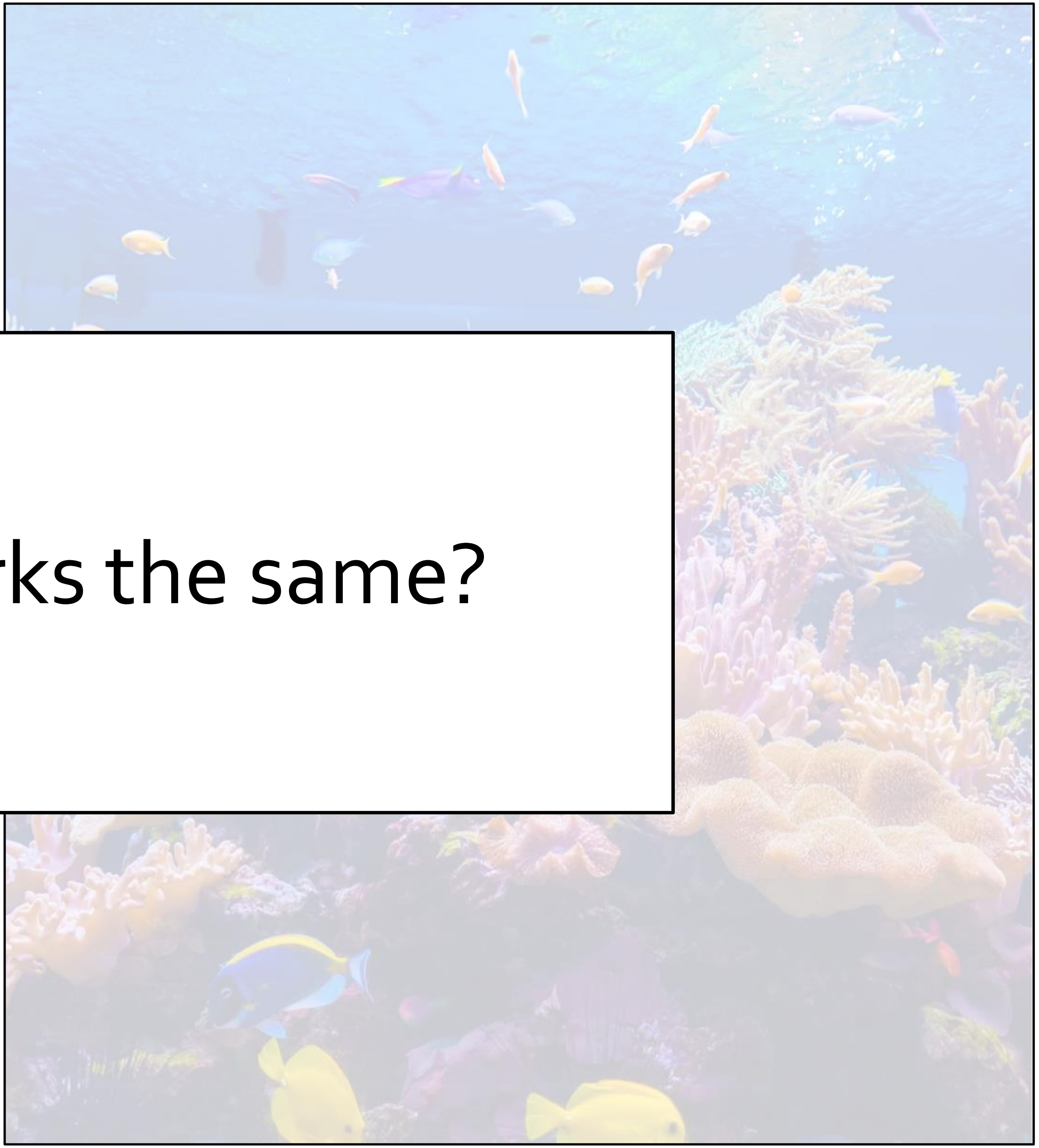
Quantifying interactions and understanding the role of species:

Direct interactions (degree)

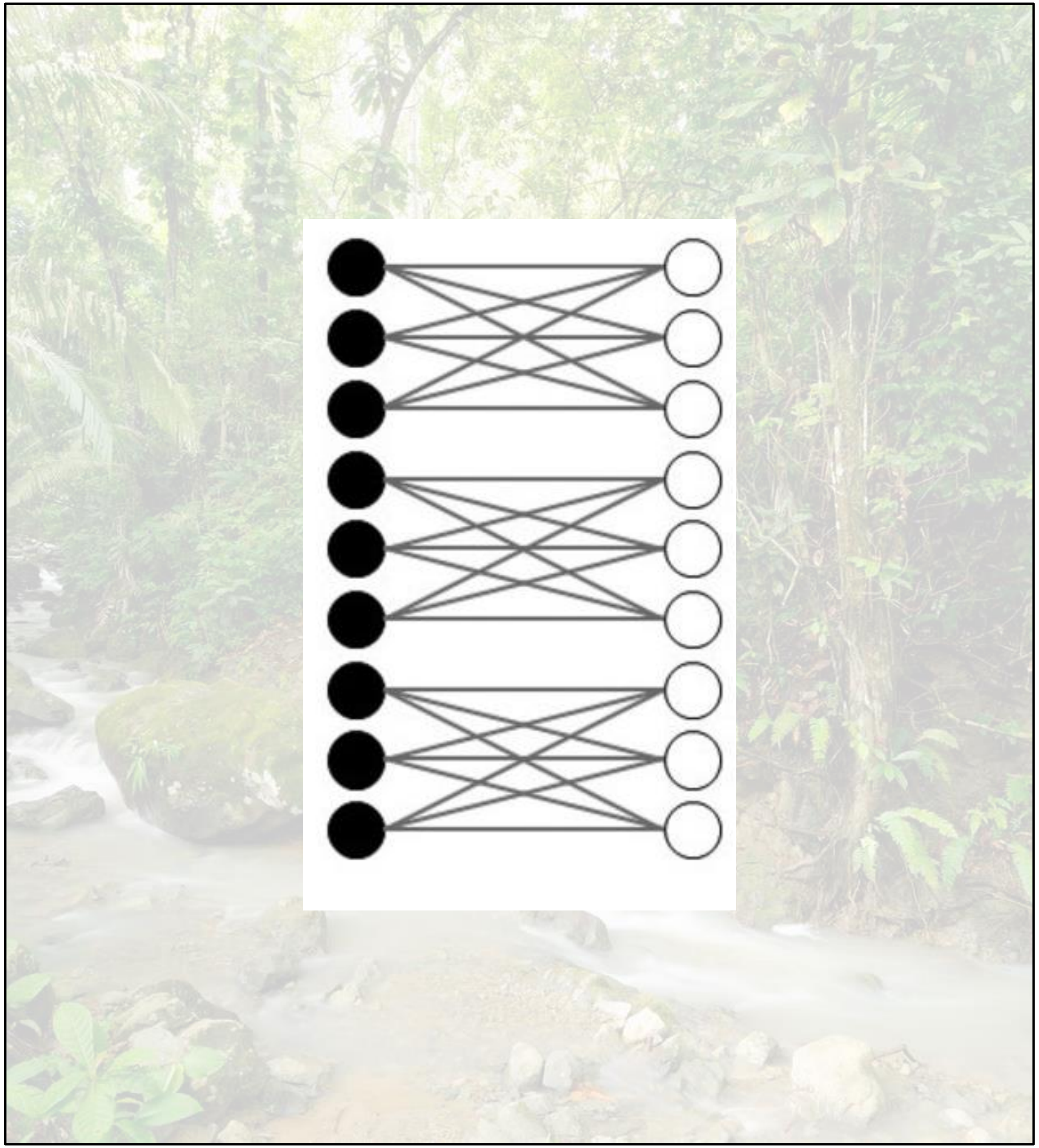
Indirect interactions through shortest pathways (closeness)

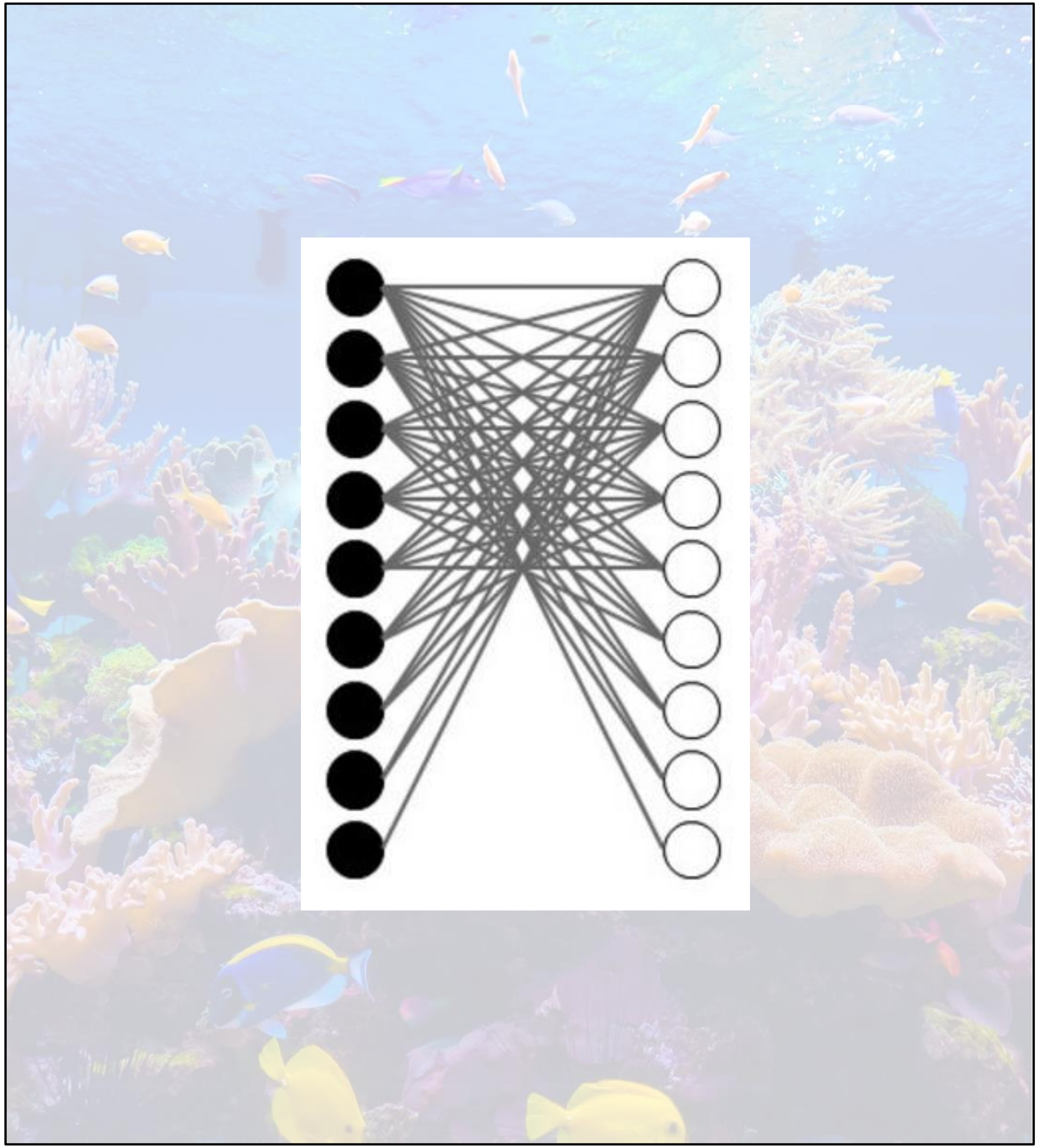
Indirect interactions through all possible pathways (Katz)





Are all networks the same?





The role of species in networks

BIO365 – Ecological Networks

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