Topological patterns in ecological networks

BIO365 – Ecological Networks

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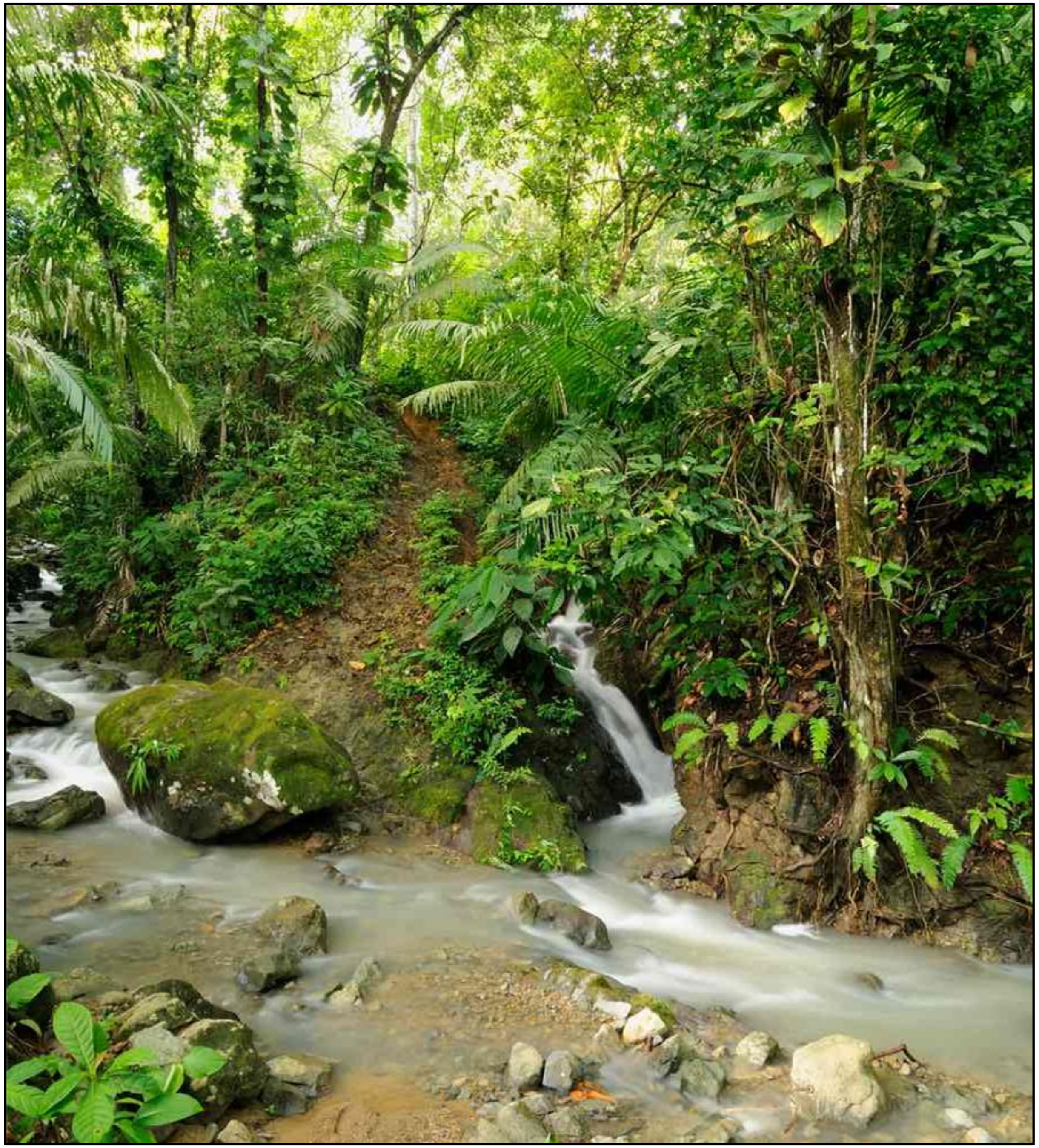
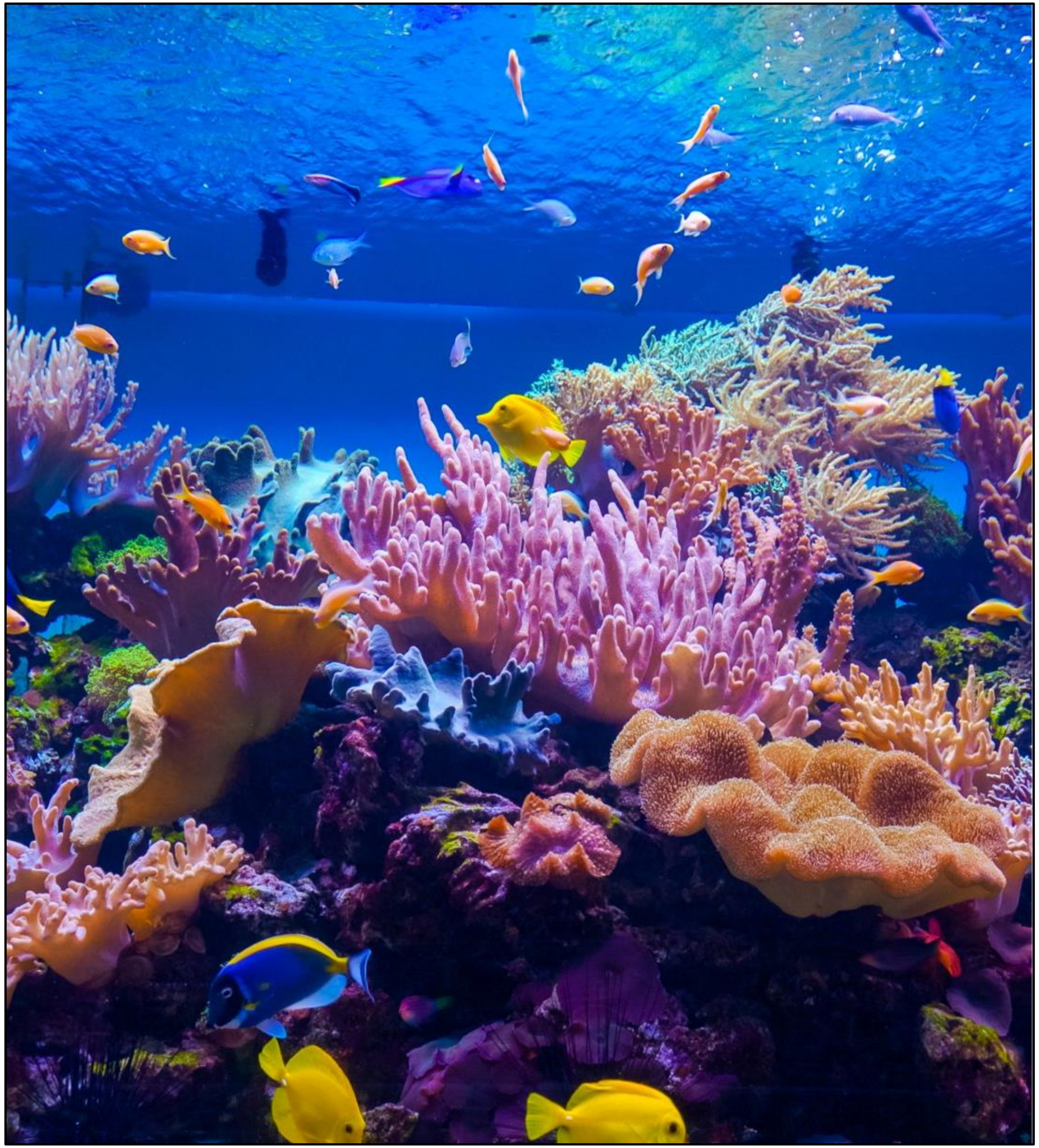
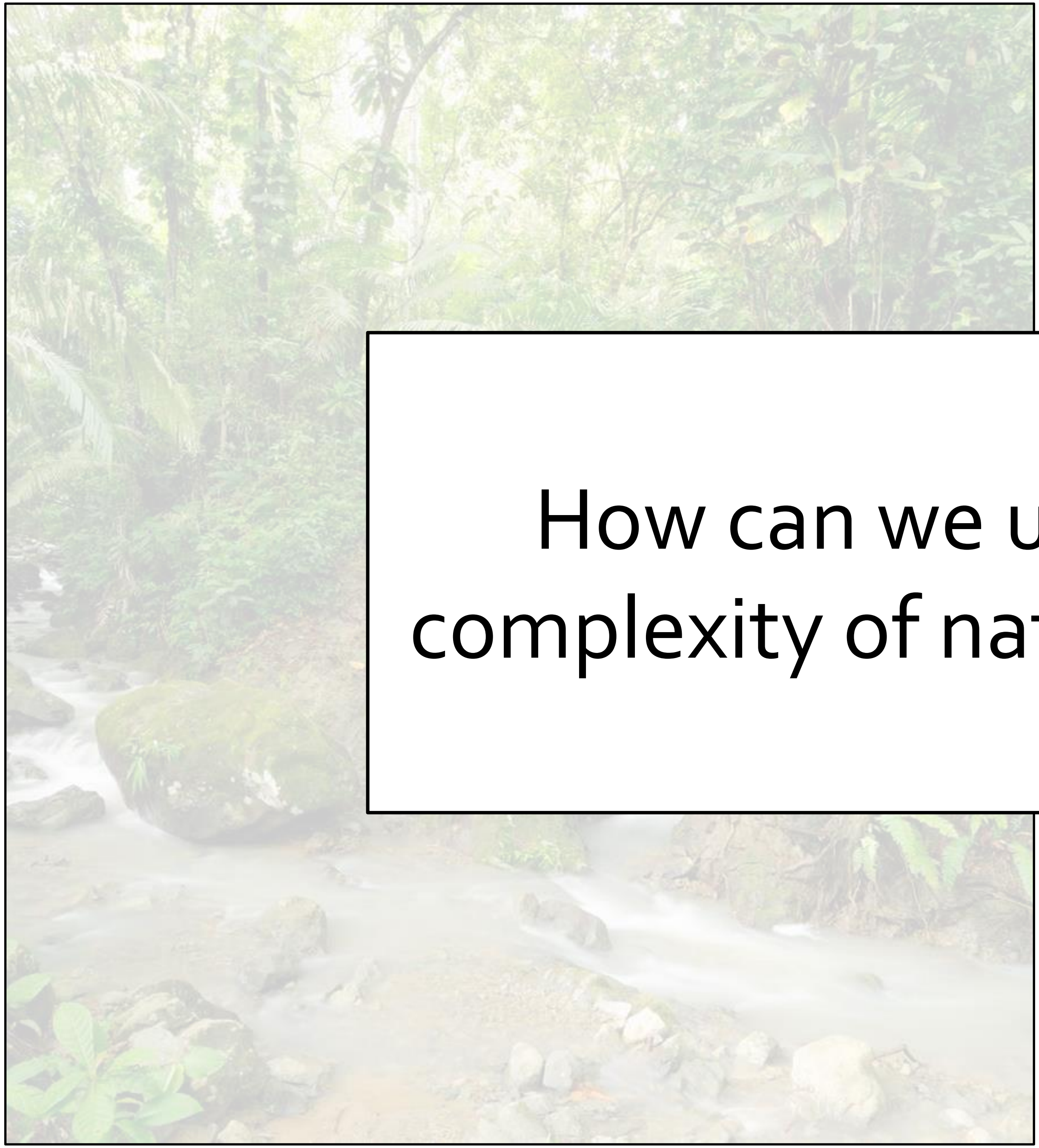
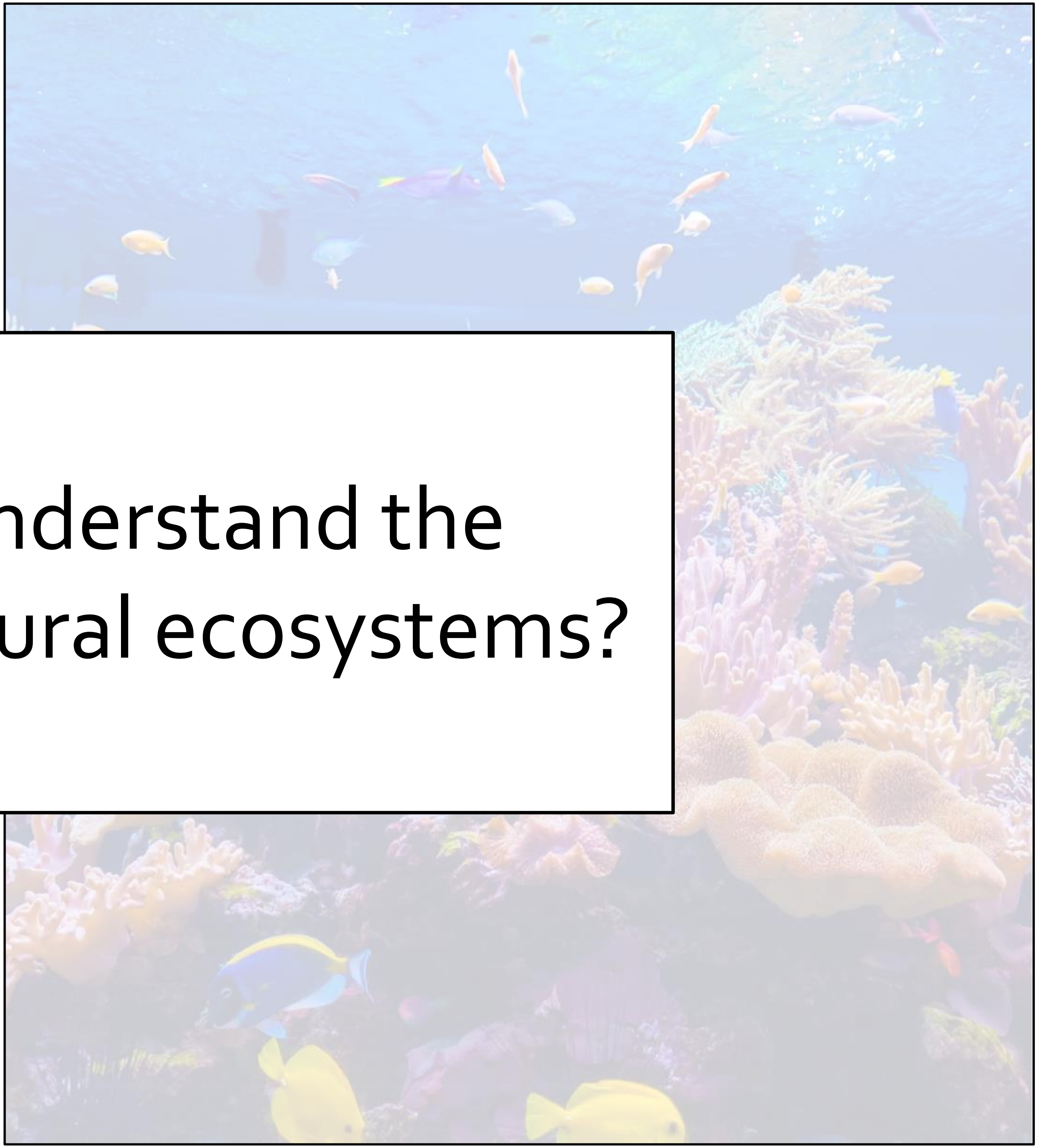







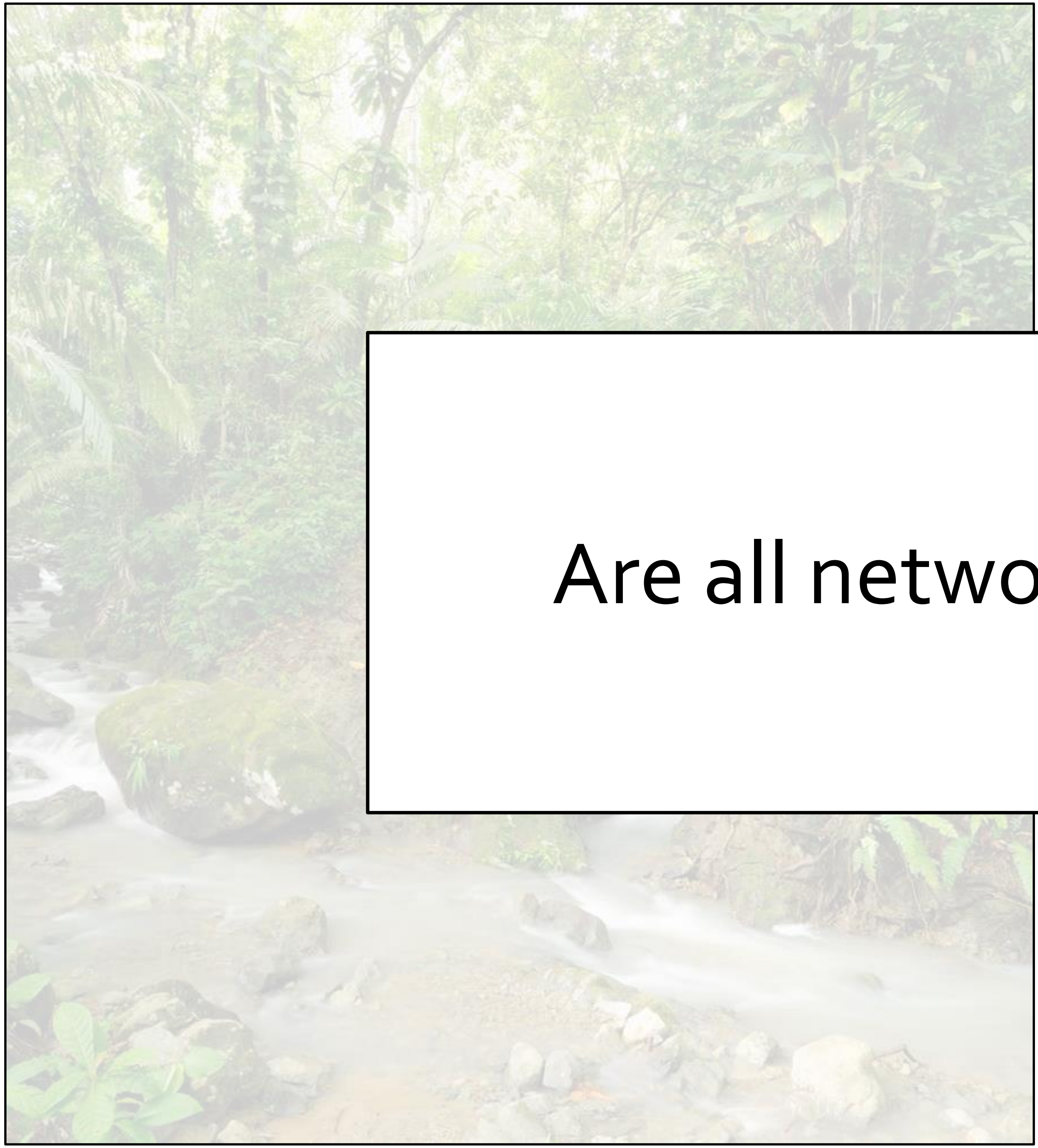
How can we understand the complexity of natural ecosystems?

Quantifying interactions and understanding the role of species:

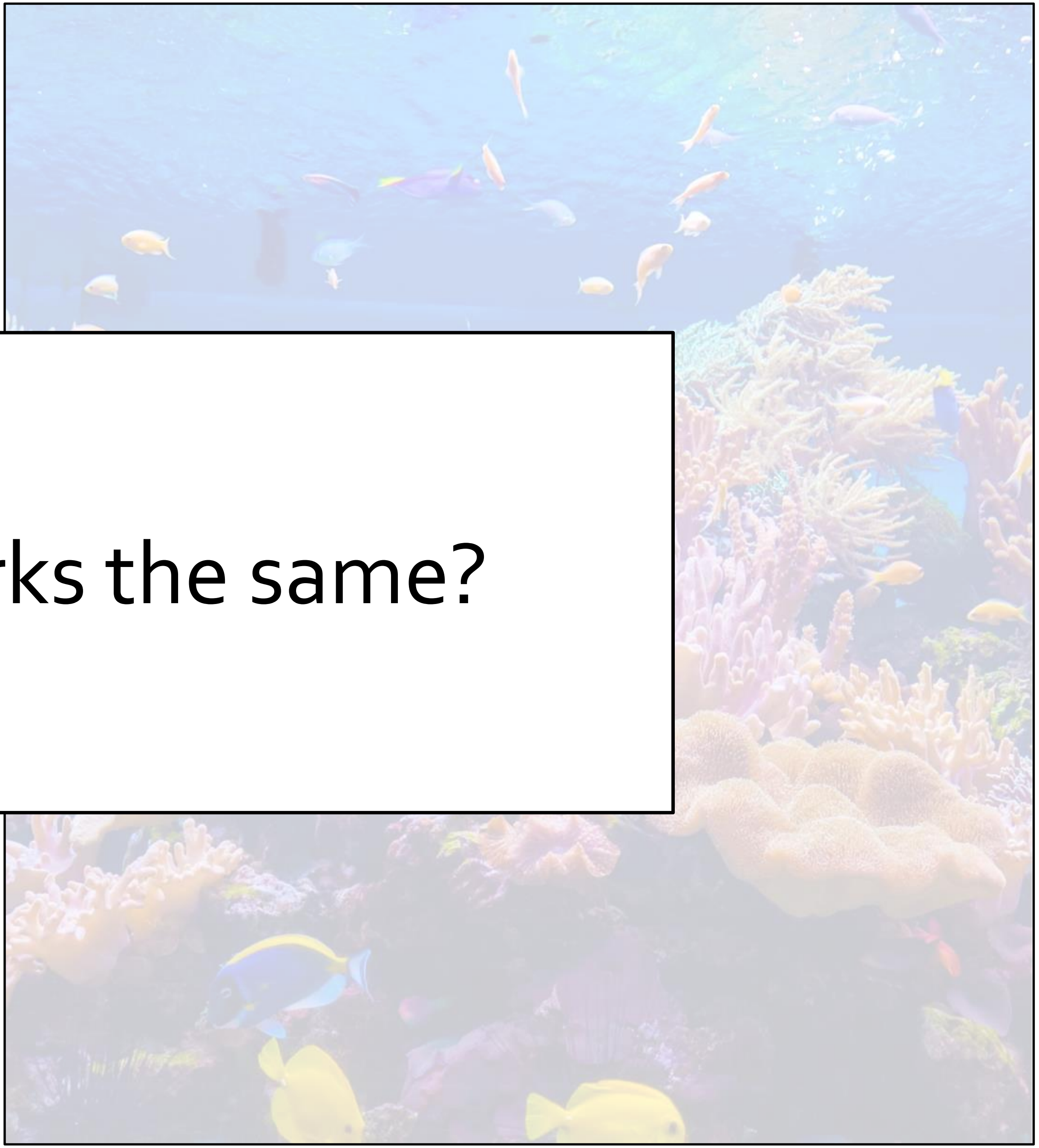
Direct interactions (degree)

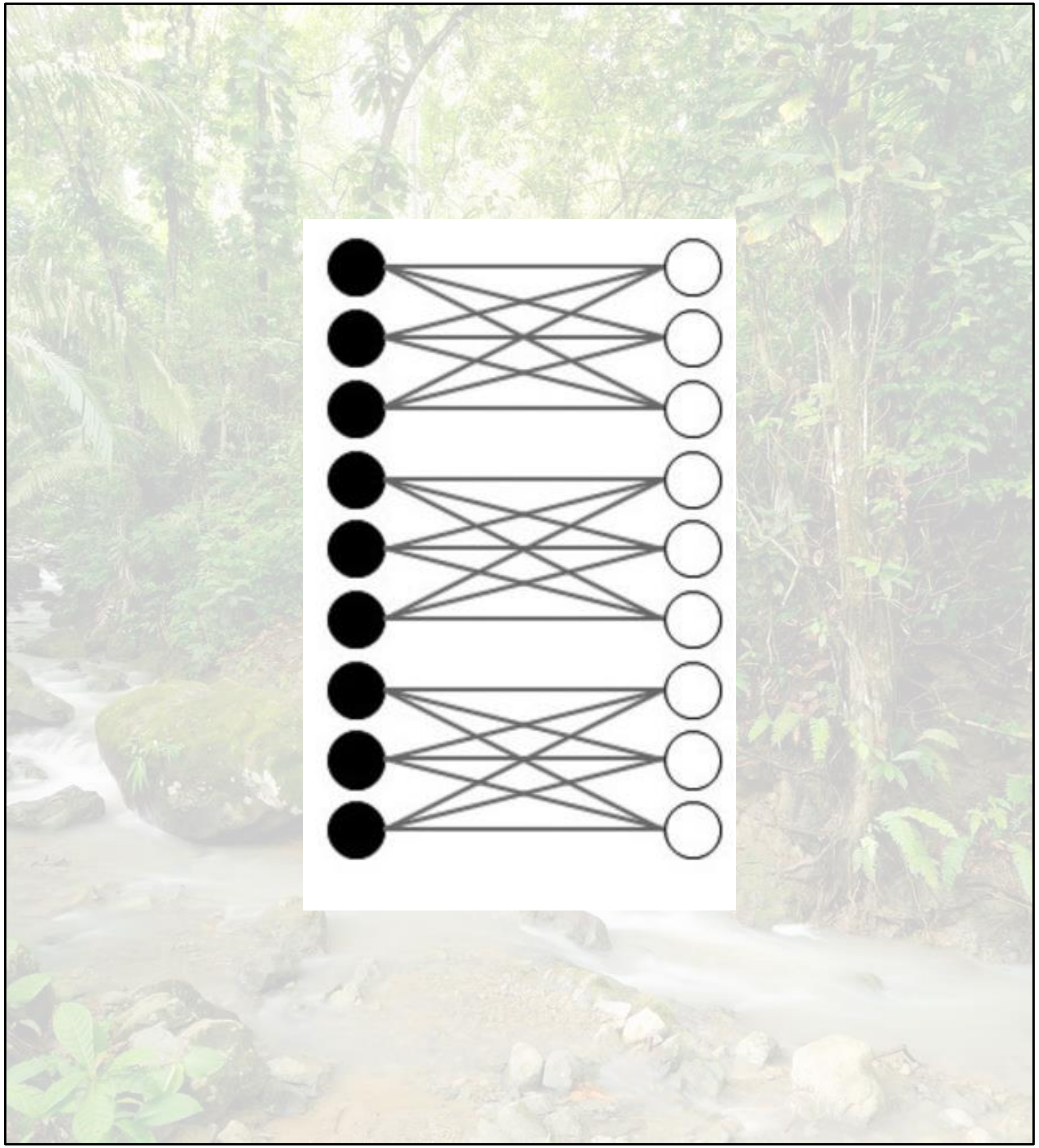
Indirect interactions through shortest pathways (closeness)

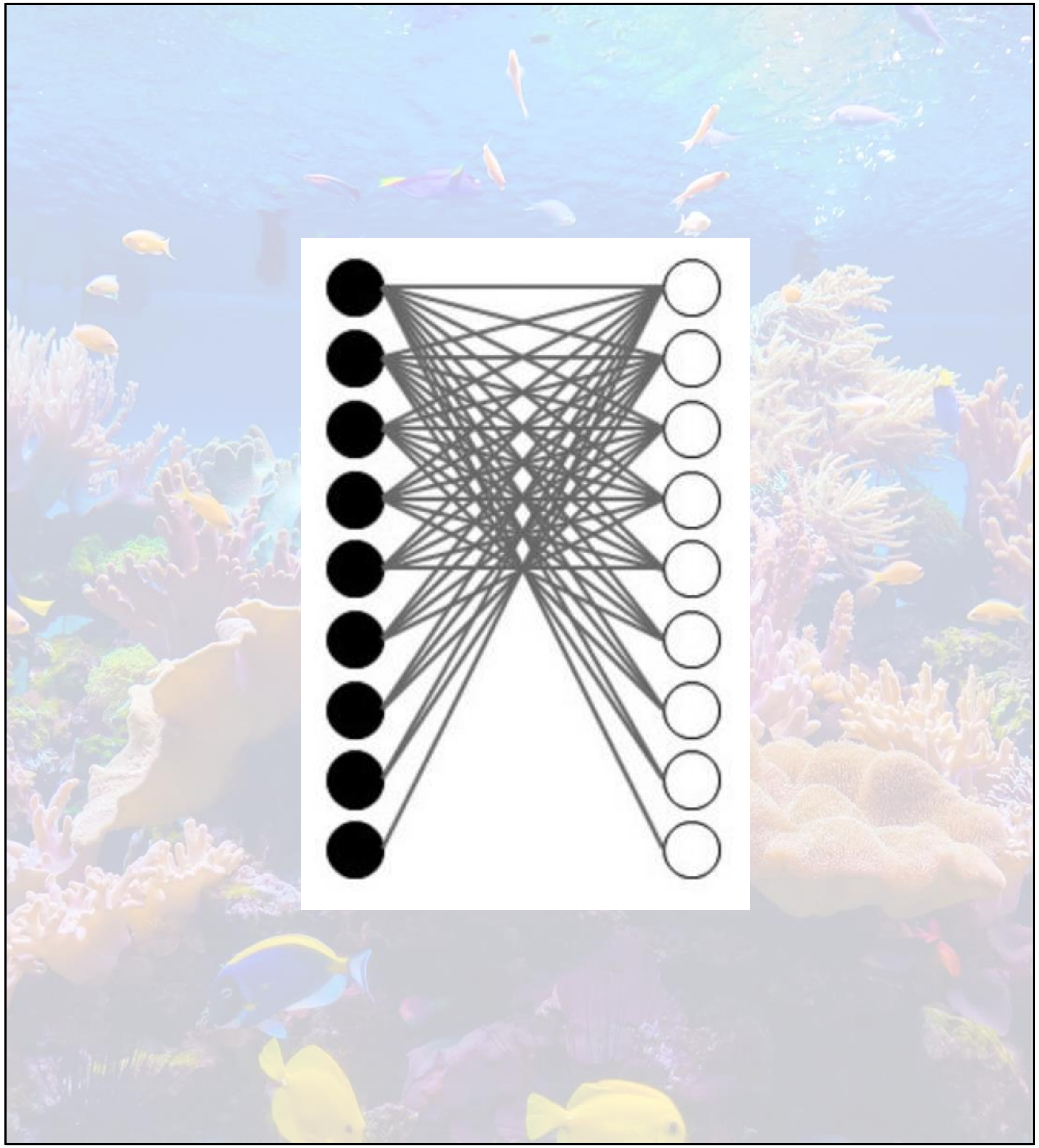
Indirect interactions through all possible pathways (Katz)

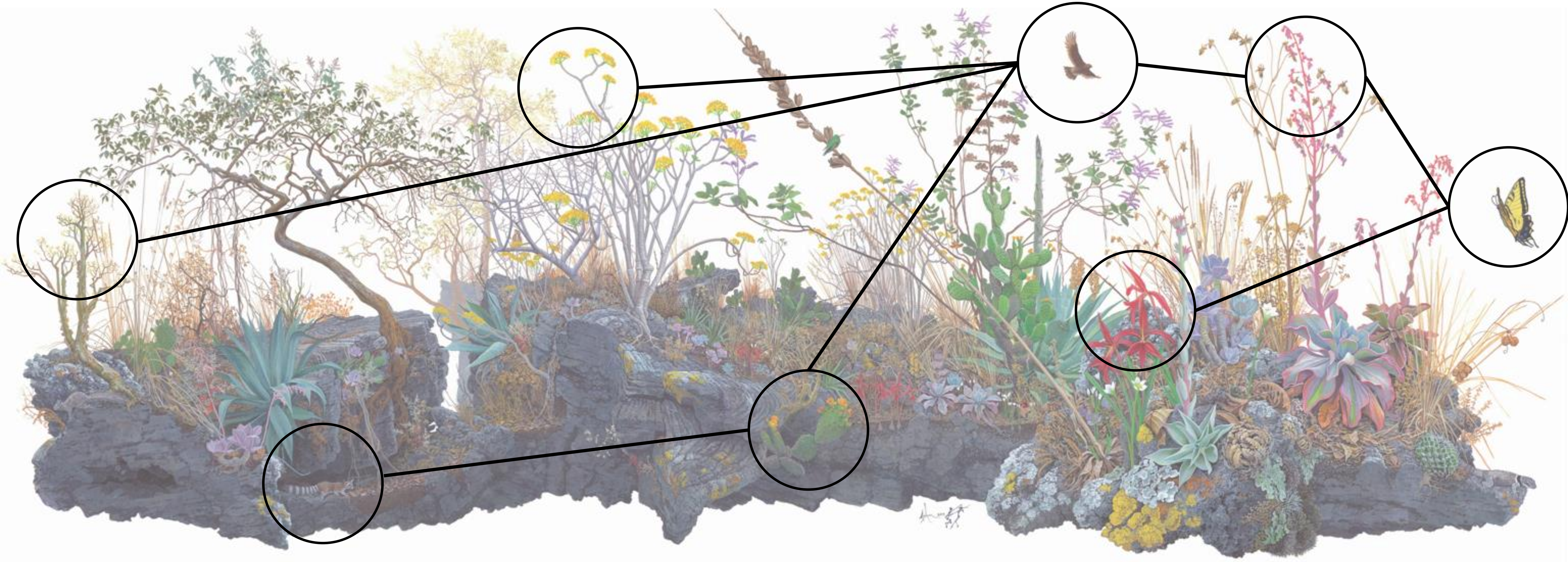


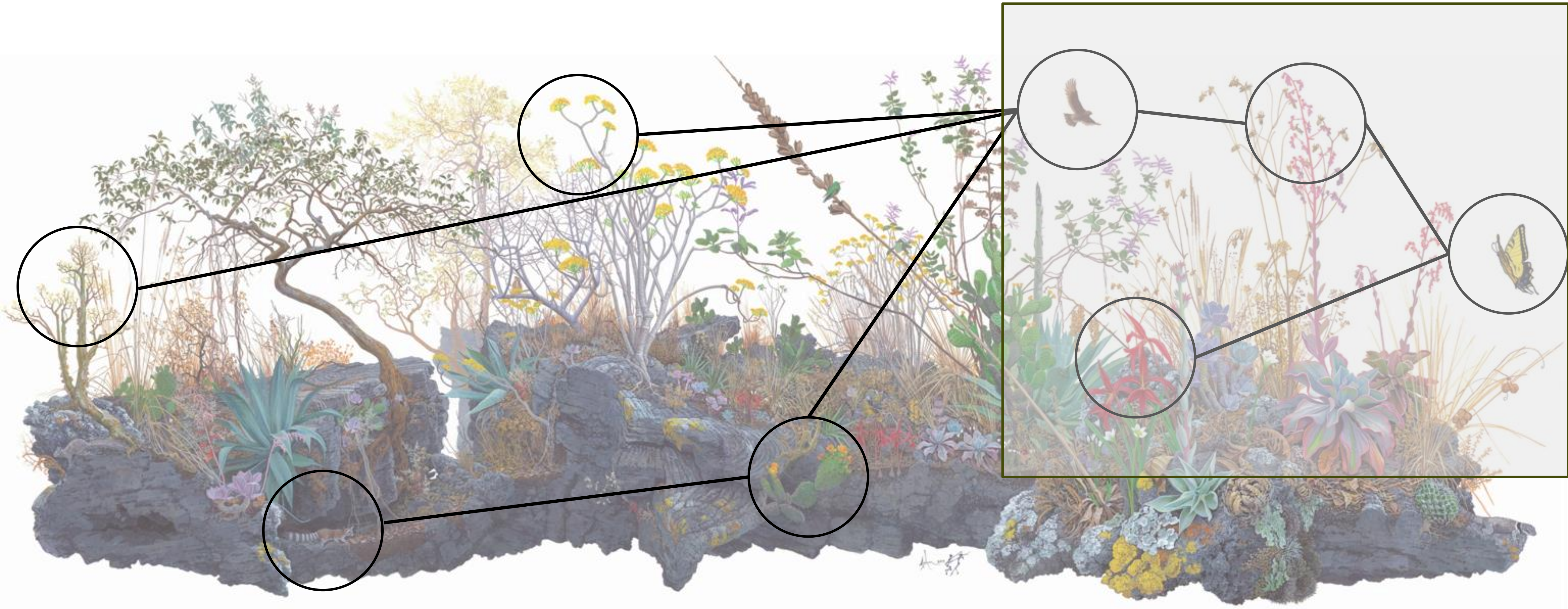
Are all networks the same?

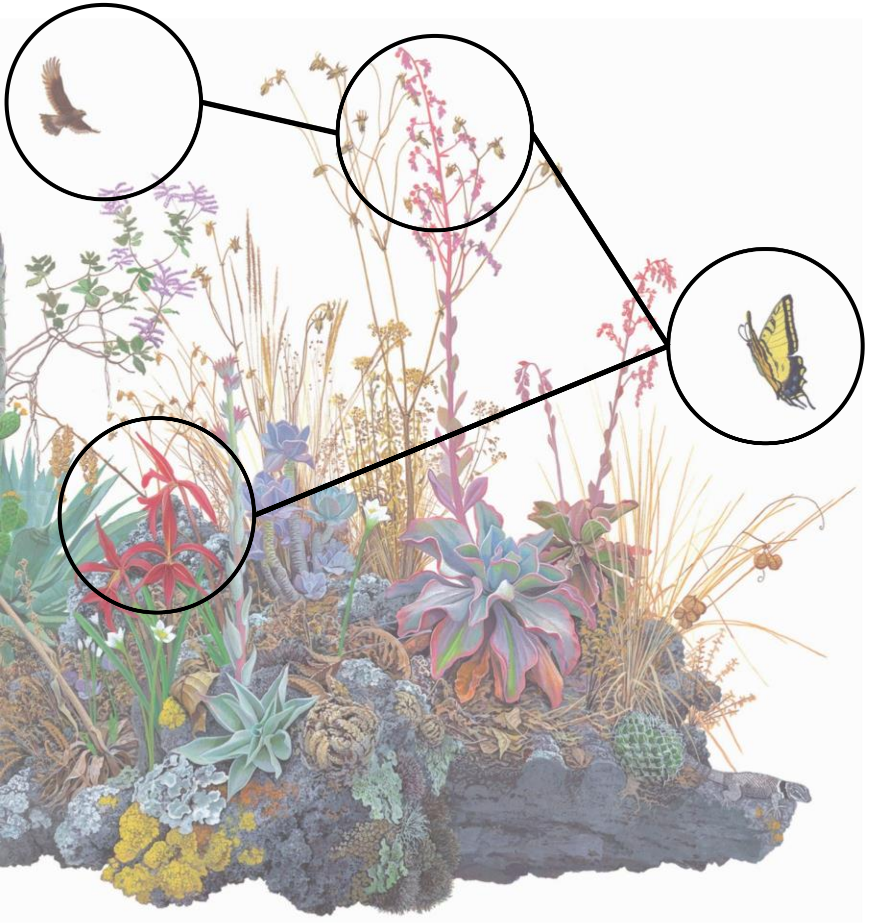






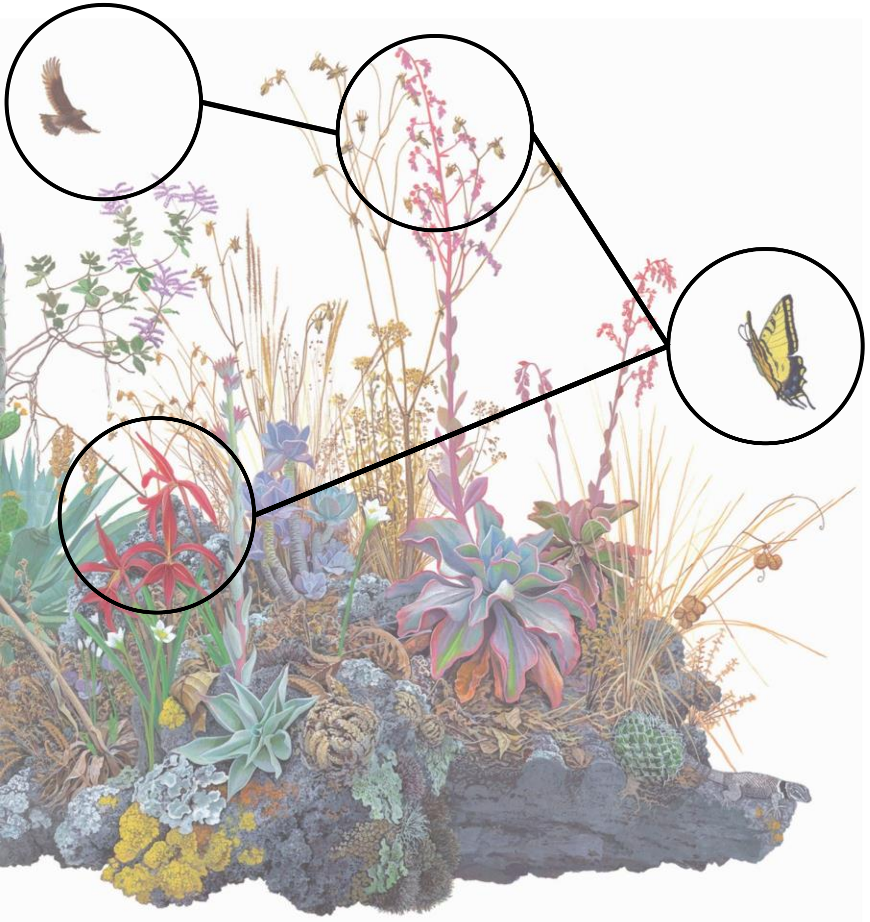






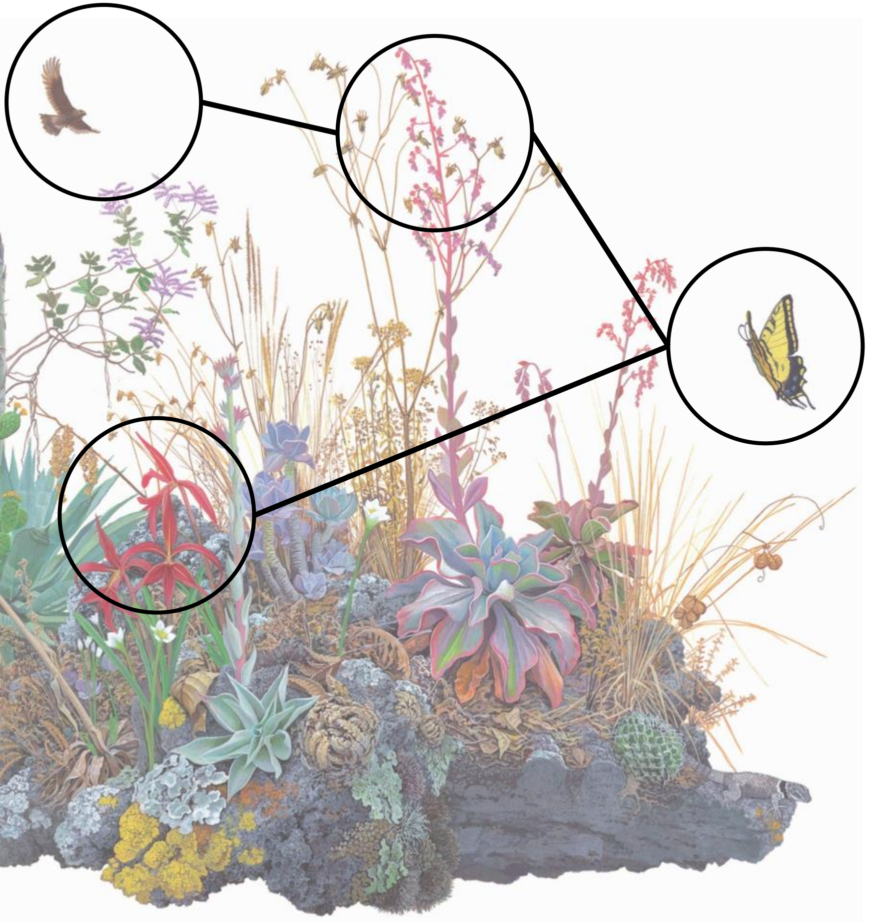
$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

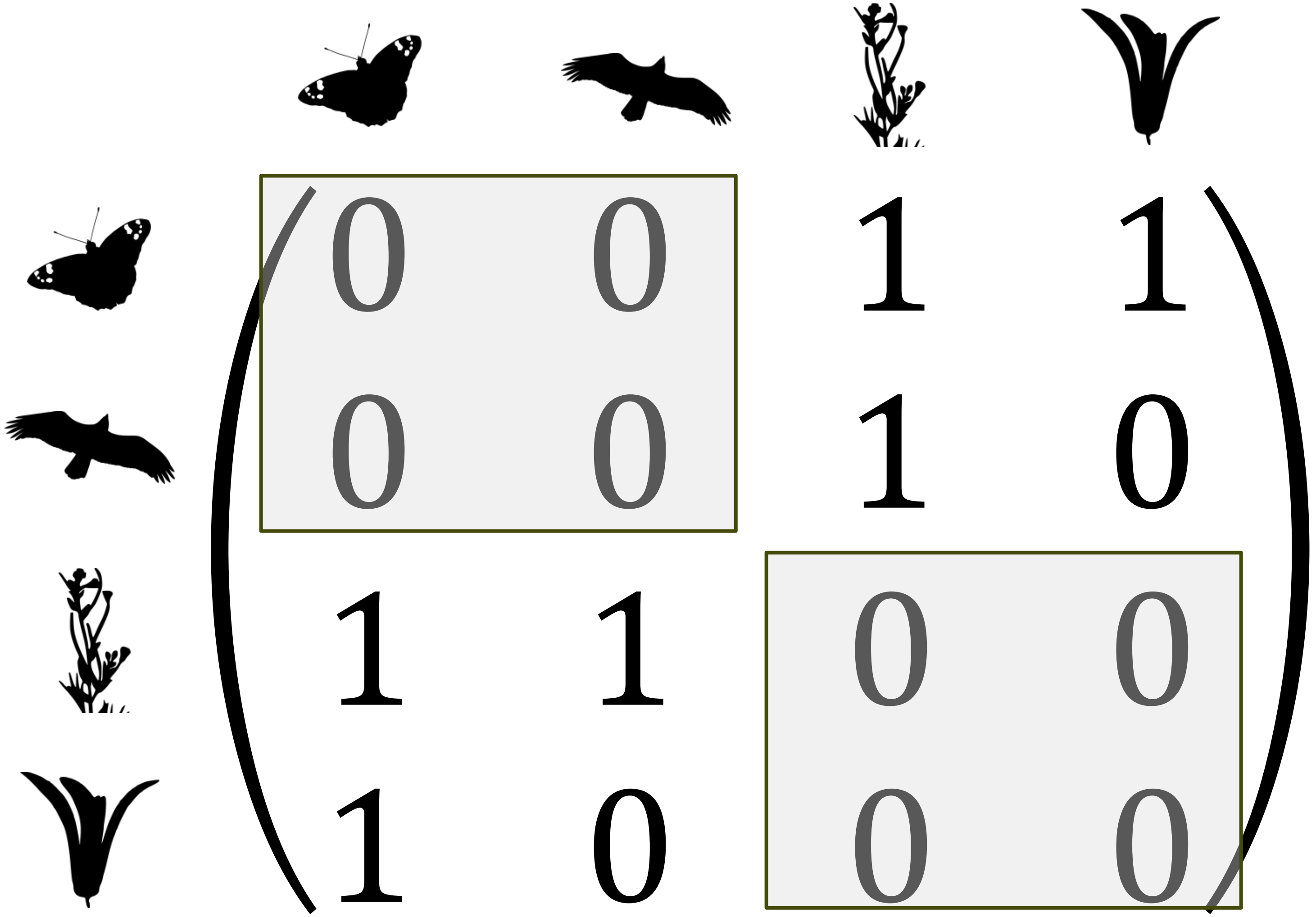
Networks can be represented as interaction matrices



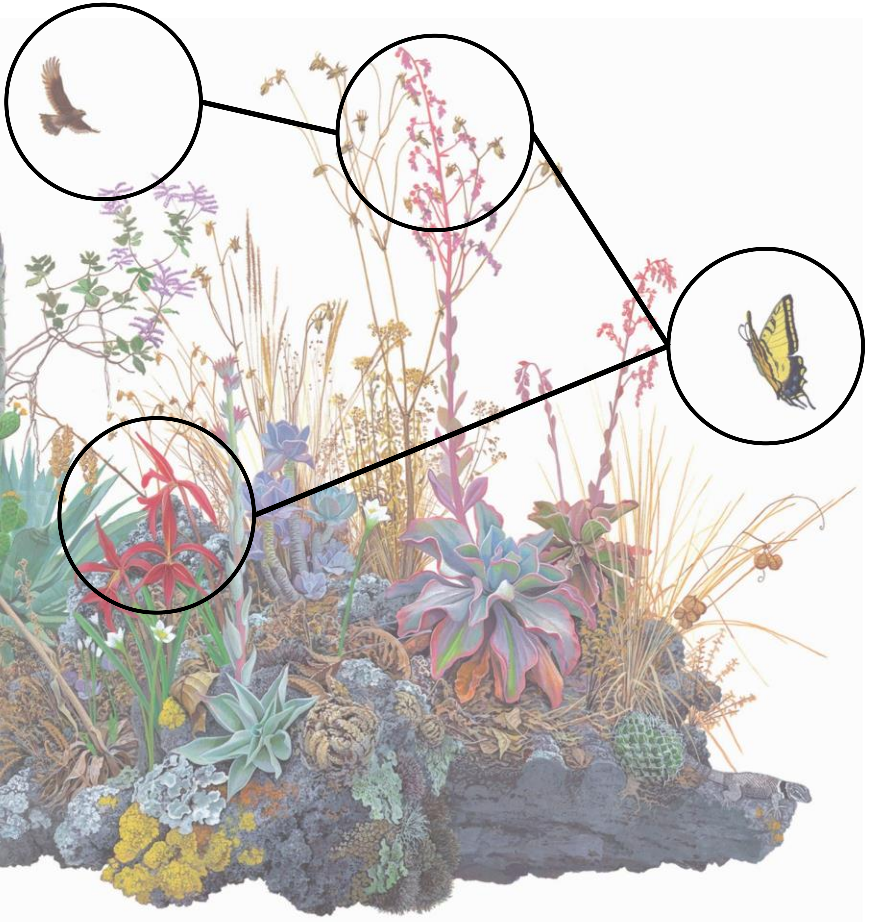
$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Adjacency matrix: all species in rows and columns



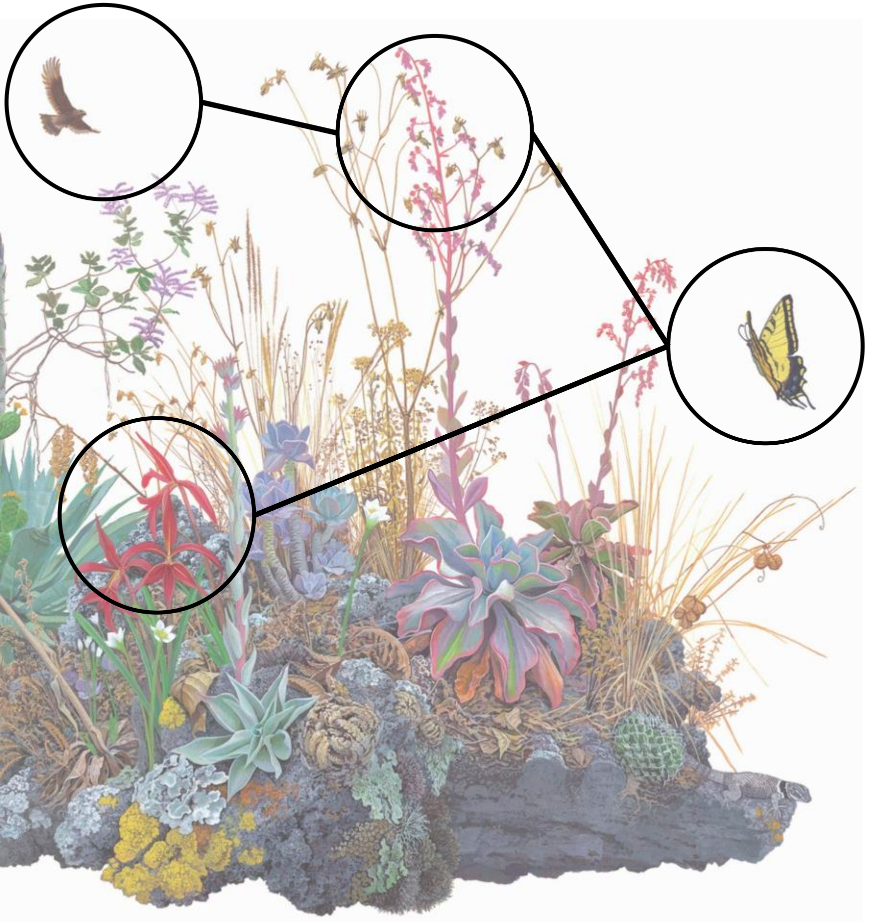


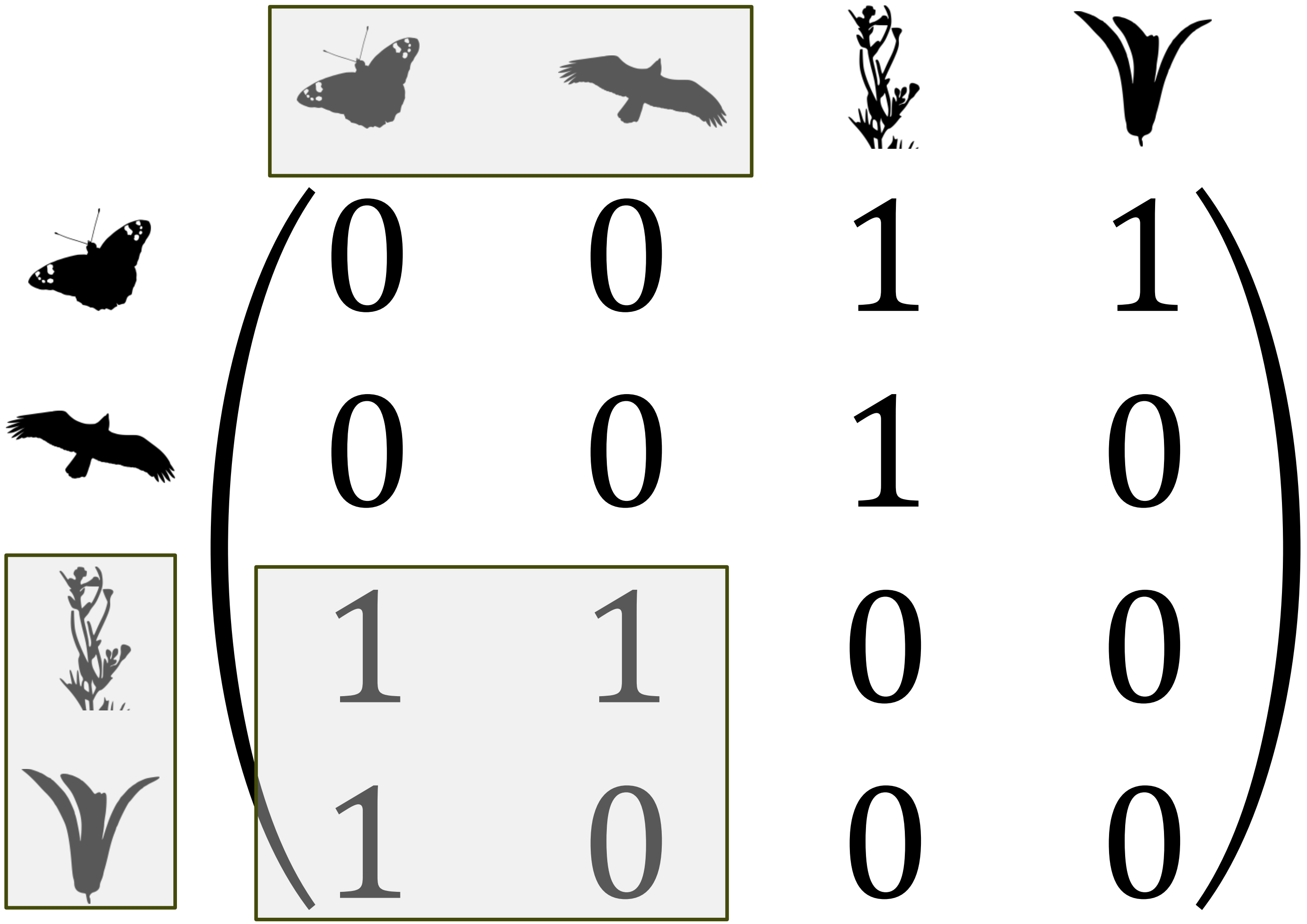
Adjacency matrix: all species in rows and columns



$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

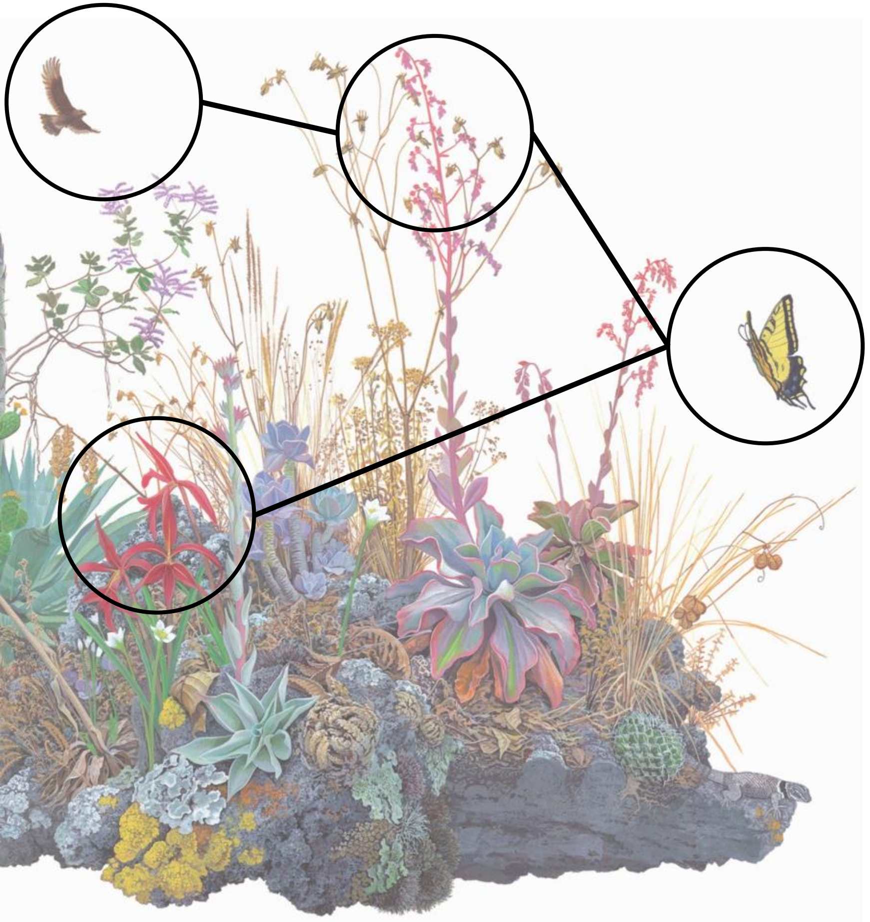
Incidence/Biadjacency matrix: species from one set in rows and from another in columns

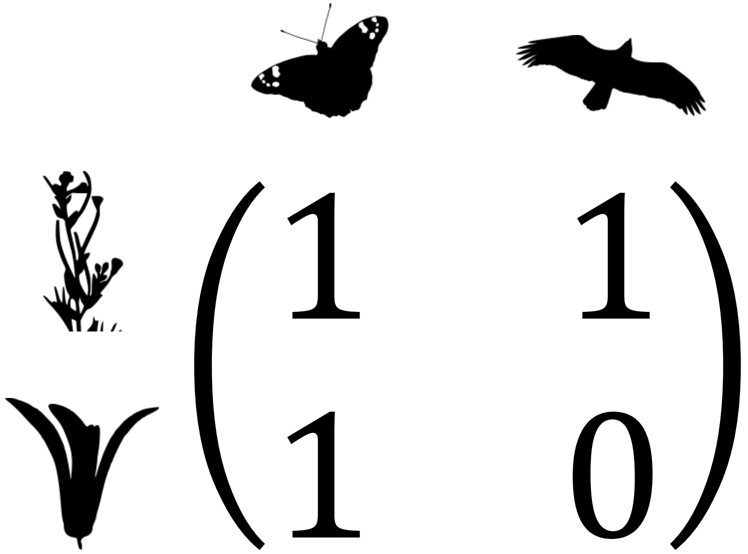




$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

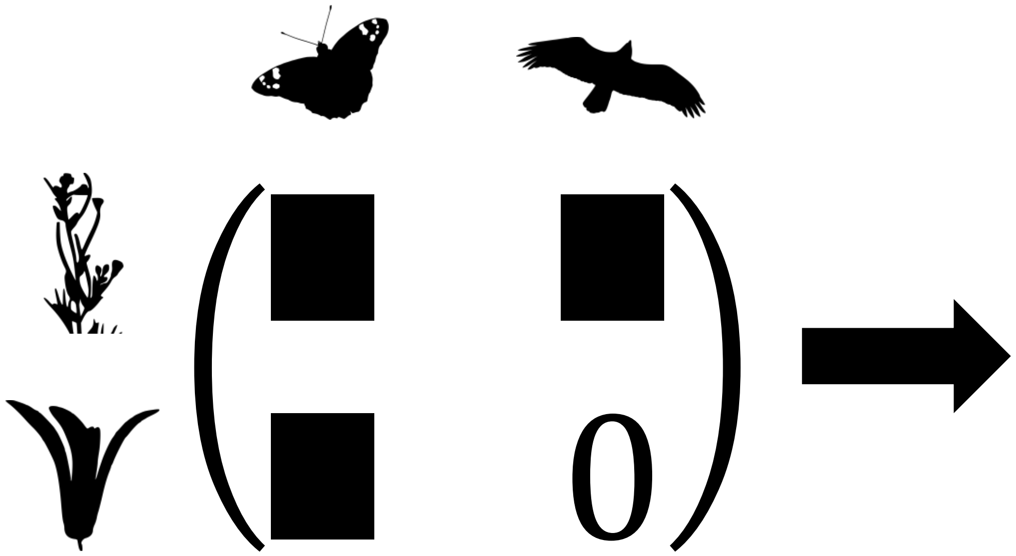
Incidence/Biadjacency matrix: species from one set in rows and from another in columns



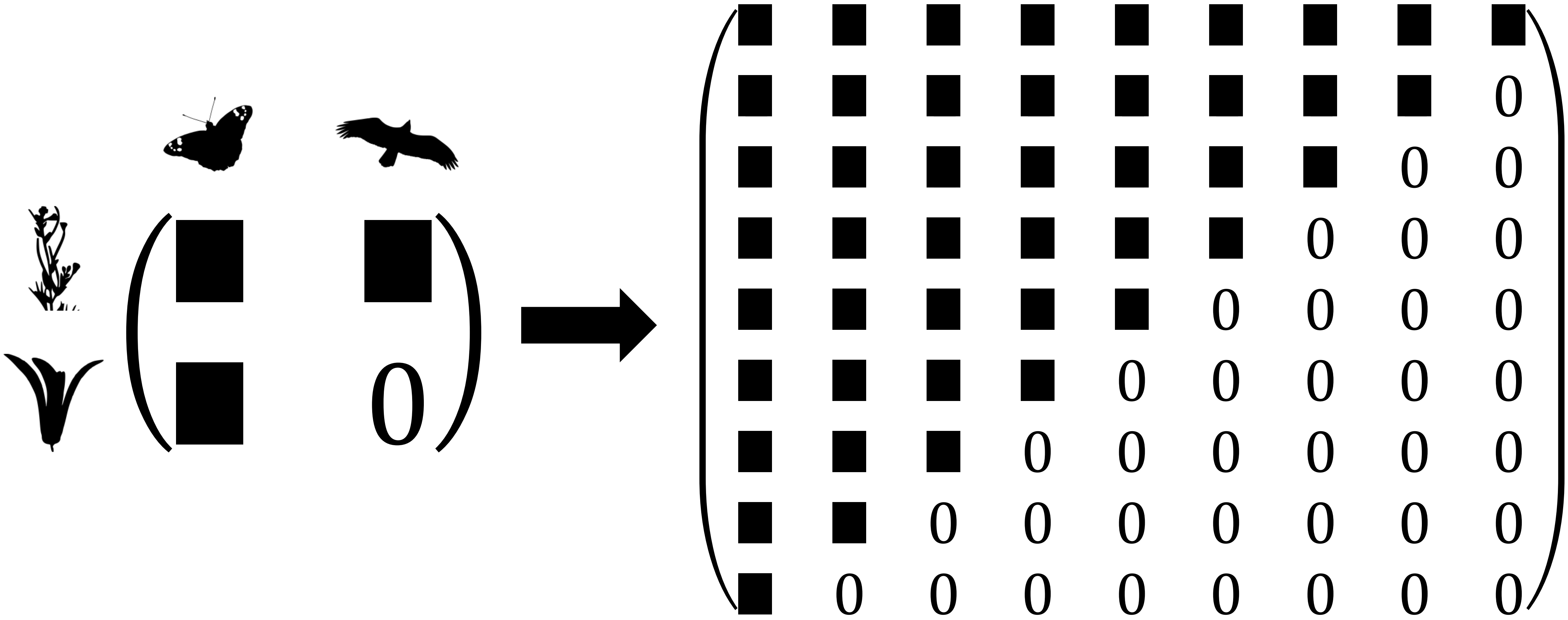


$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

Incidence/Biadjacency matrix: species from one set in rows and from another in columns



Scaling up to more plant and animal species: can we see any pattern in the incidence matrix?



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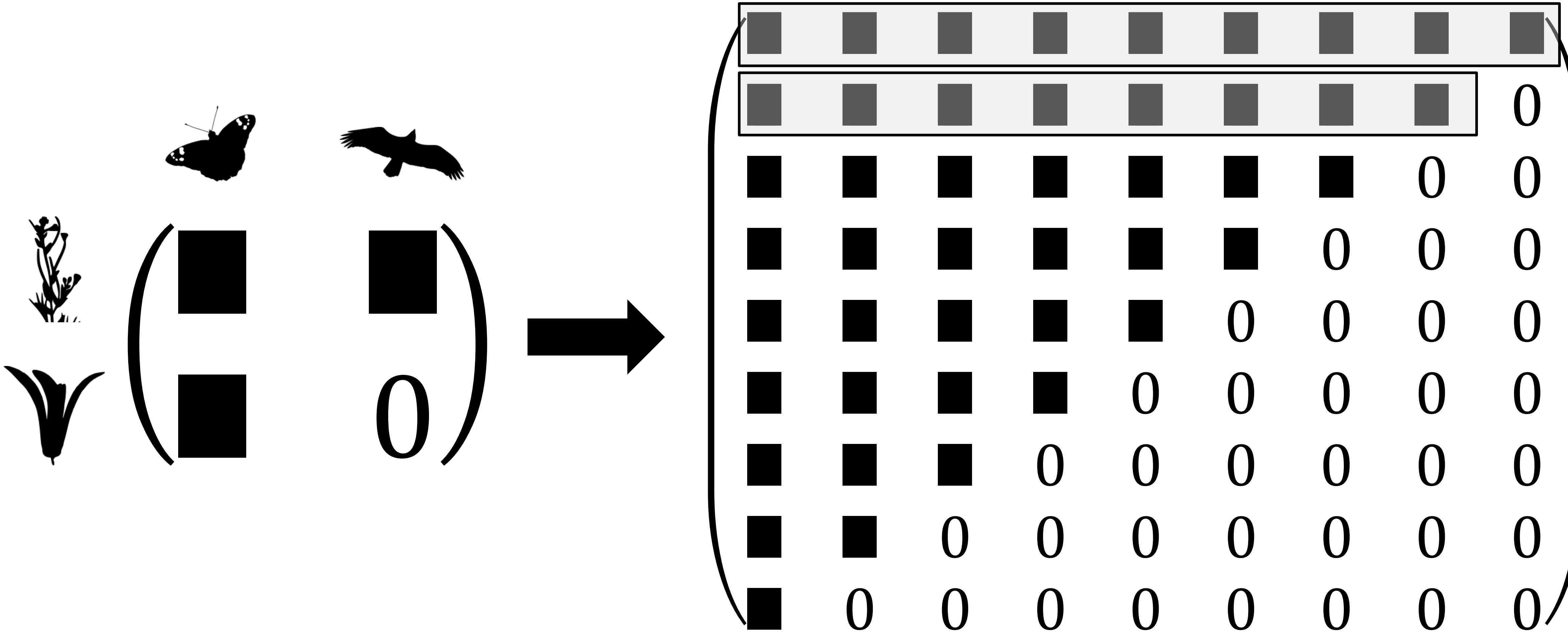
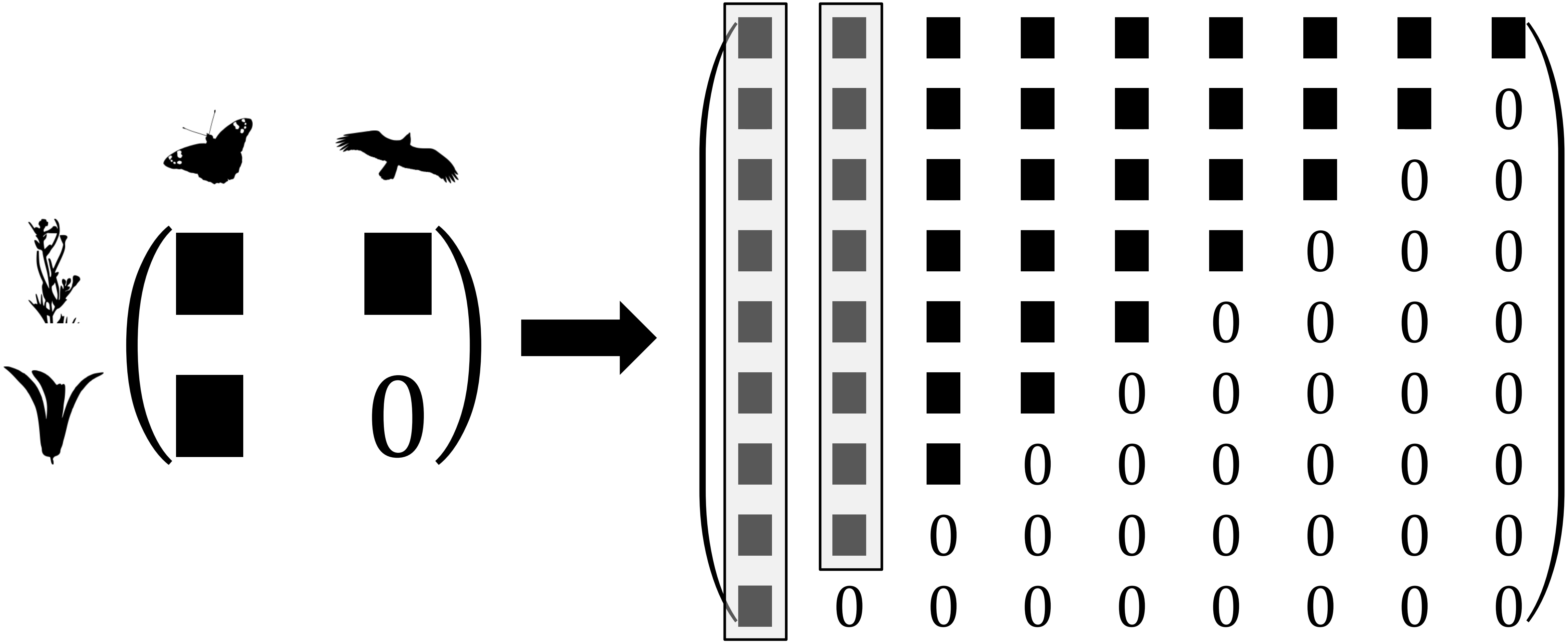
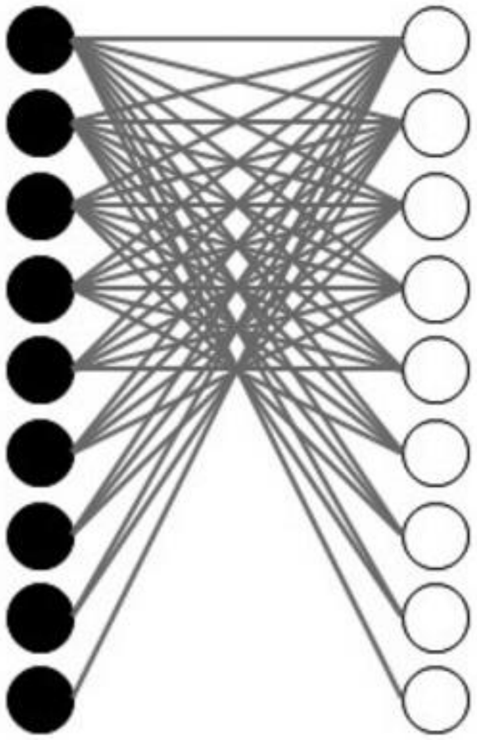


Interactions of plants with a lower degree are contained within the interactions of plants with a higher degree

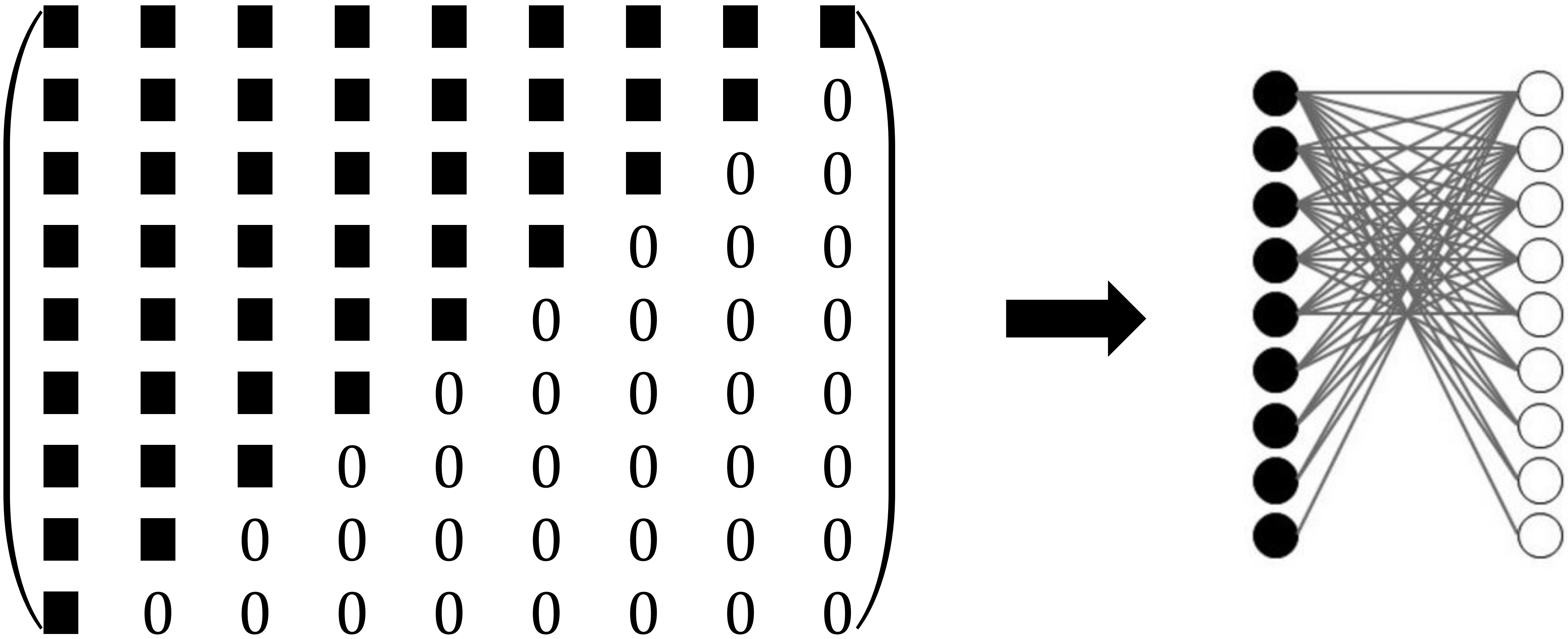


Interactions of animals with a lower degree are contained within the interactions of animals with a higher degree

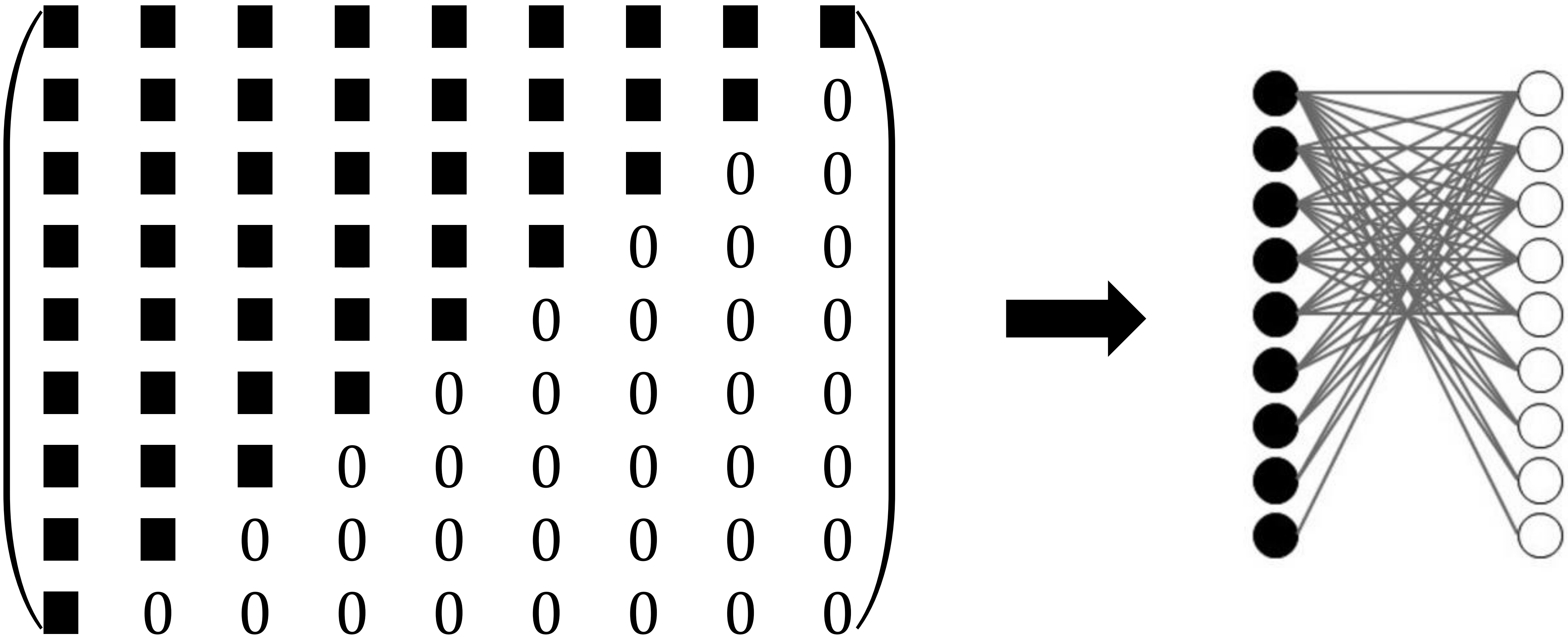
■	■	■	■	■	■	■	■	■
■	■	■	■	■	■	■	■	0
■	■	■	■	■	■	■	0	0
■	■	■	■	■	■	0	0	0
■	■	■	■	■	0	0	0	0
■	■	■	■	0	0	0	0	0
■	■	■	0	0	0	0	0	0
■	■	0	0	0	0	0	0	0
■	0	0	0	0	0	0	0	0



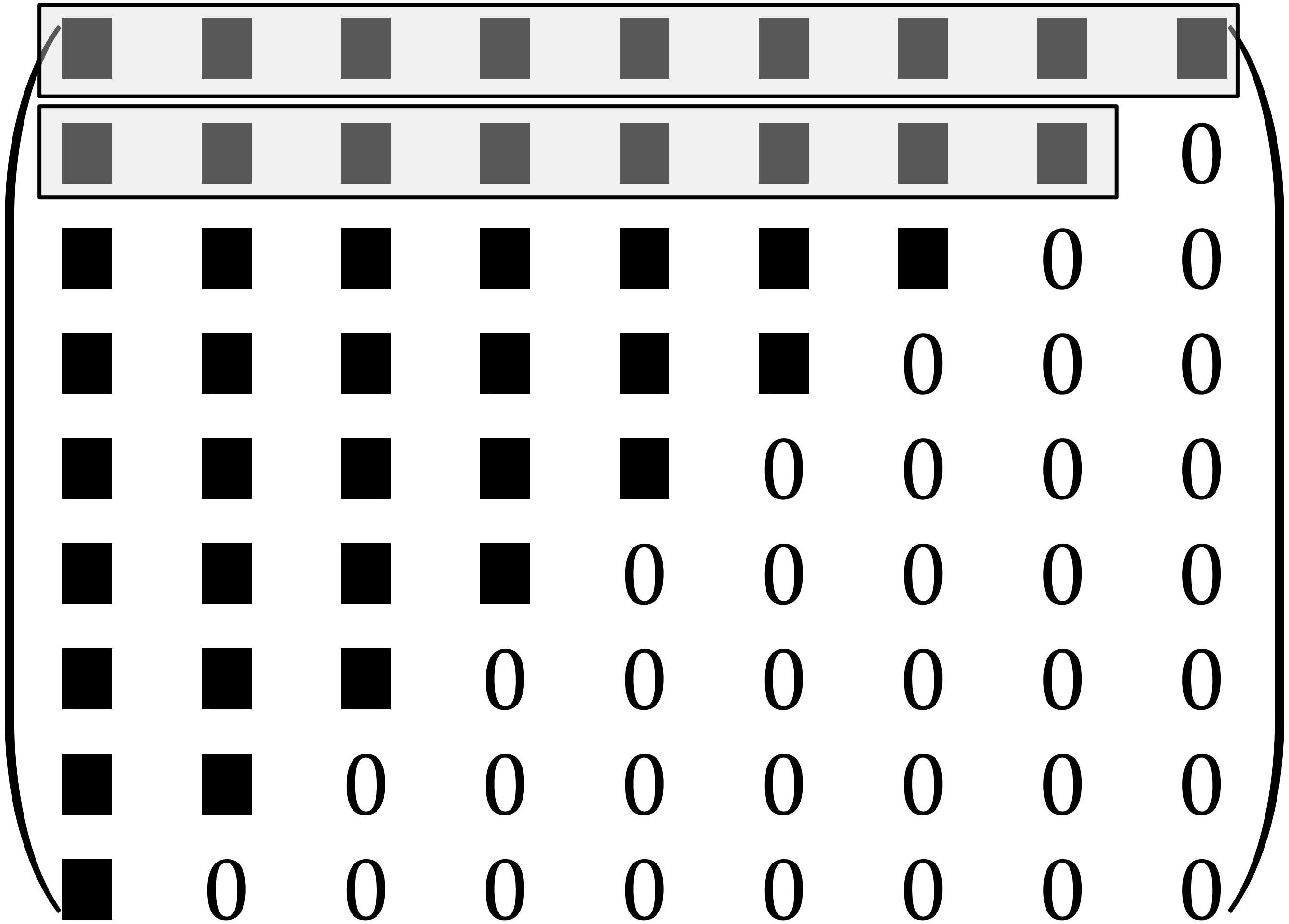
Can we see any pattern in the incidence matrix? Nestedness!



Nestedness: interactions of specialists are a subset of the interactions of generalists

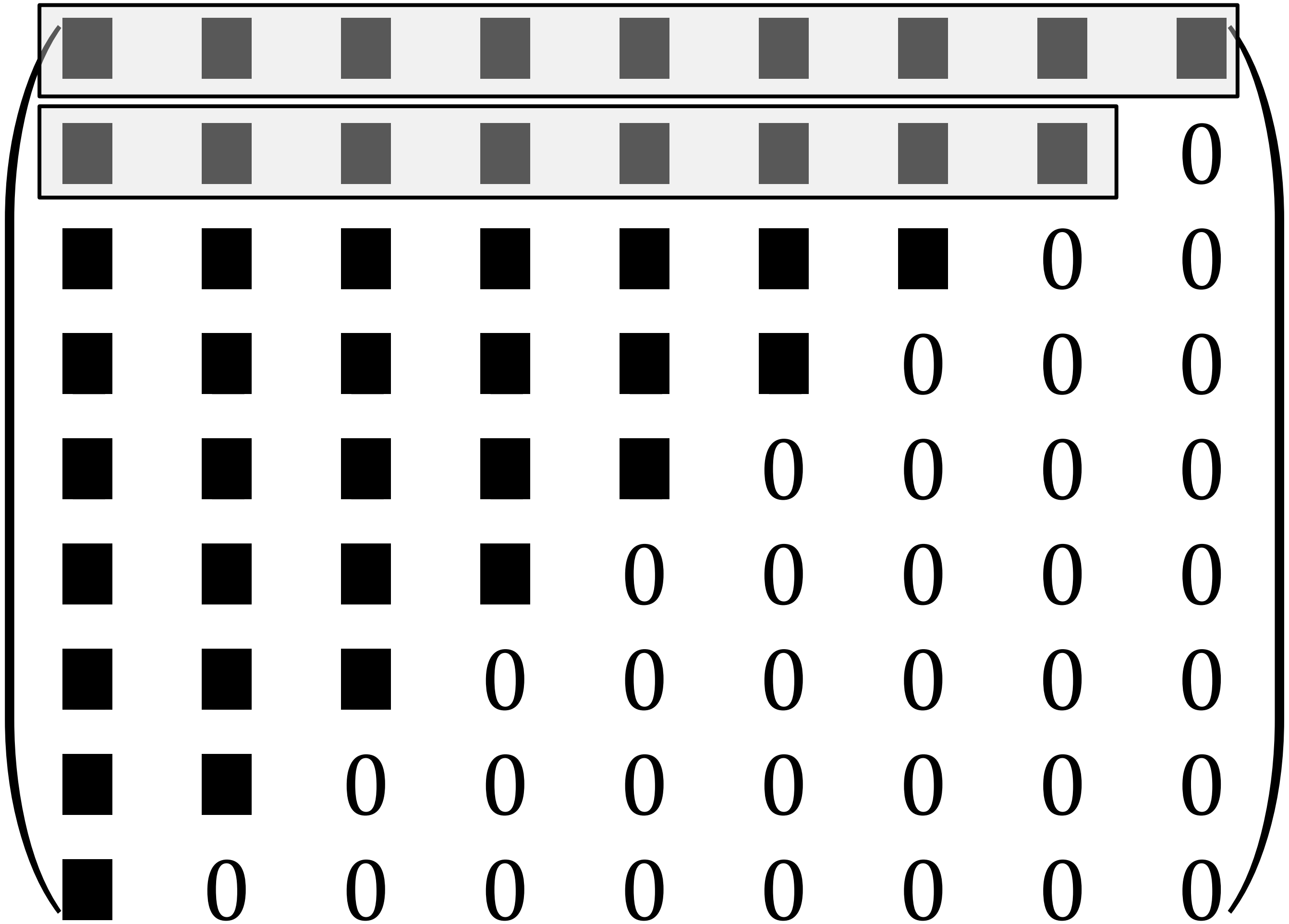


How can we quantify this pattern?



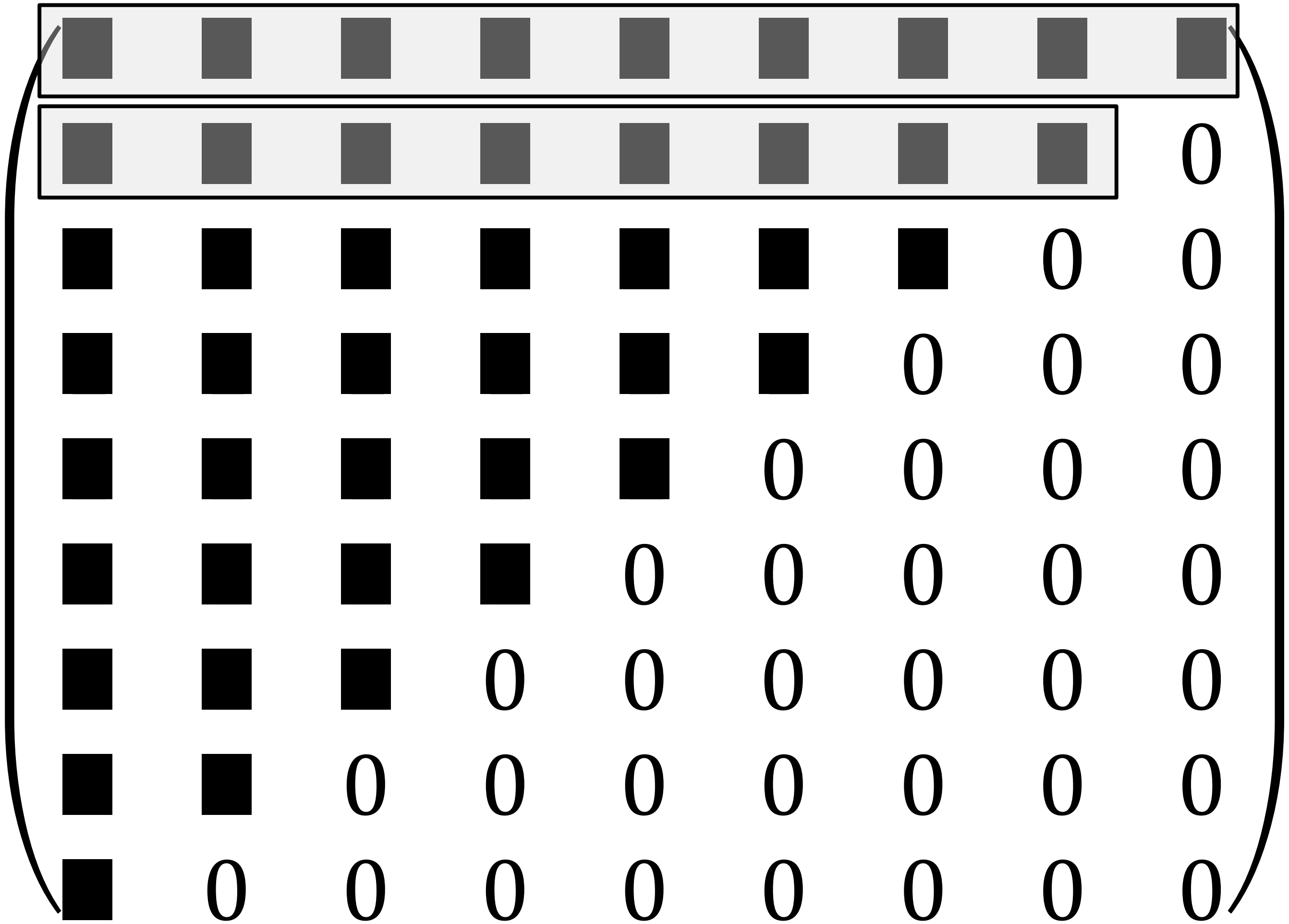
How can we quantify this pattern?

Measuring overlap between pairs of plants (rows)



$\boldsymbol{c_{ij}}$ = Number of shared interactions between row i and j

How can we quantify this pattern?
Measuring overlap between pairs of plants (rows)

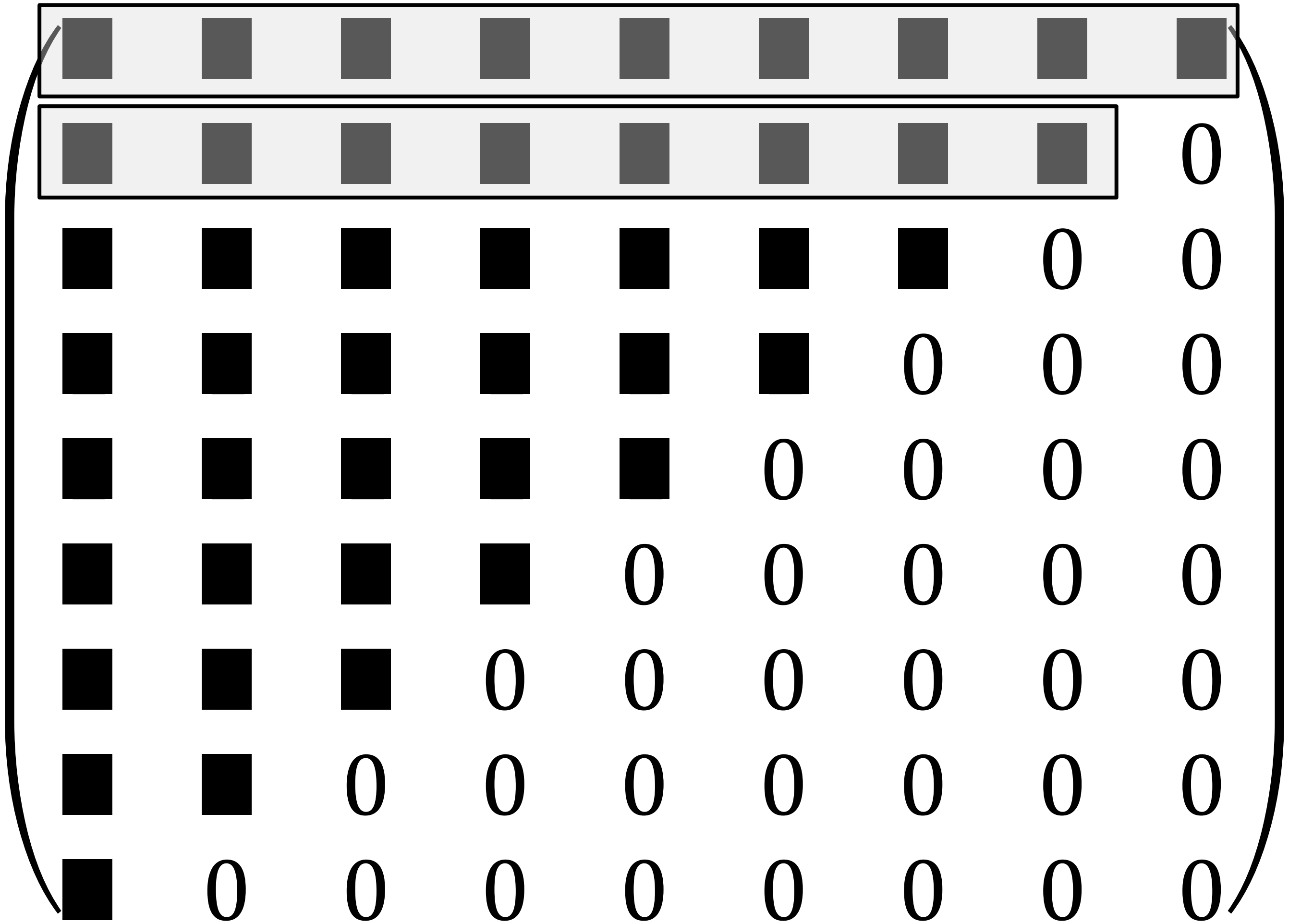


c_{ij} = Number of shared interactions between row i and j

k_i = Number of interactions in row i

k_j = Number of interactions in row j

How can we quantify this pattern?
Measuring overlap between pairs of plants (rows)



c_{ij} = Number of shared interactions between row i and j

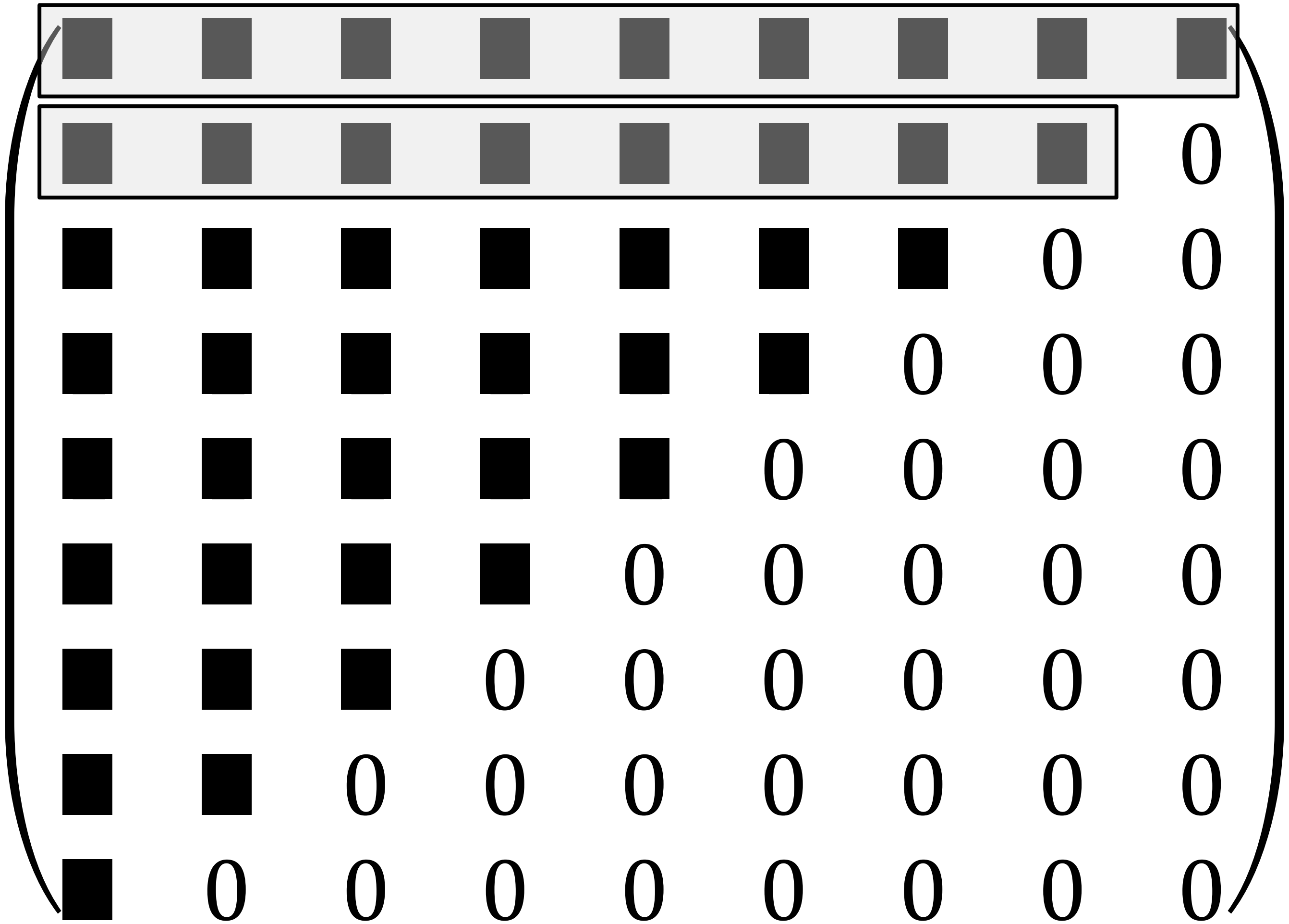
k_i = Number of interactions in row i

k_j = Number of interactions in row j

$$o_{ij} = \frac{c_{ij}}{\min(k_i, k_j)}$$

How can we quantify this pattern?

Measuring overlap between pairs of plants (rows)



c_{ij} = Number of shared interactions between row i and j

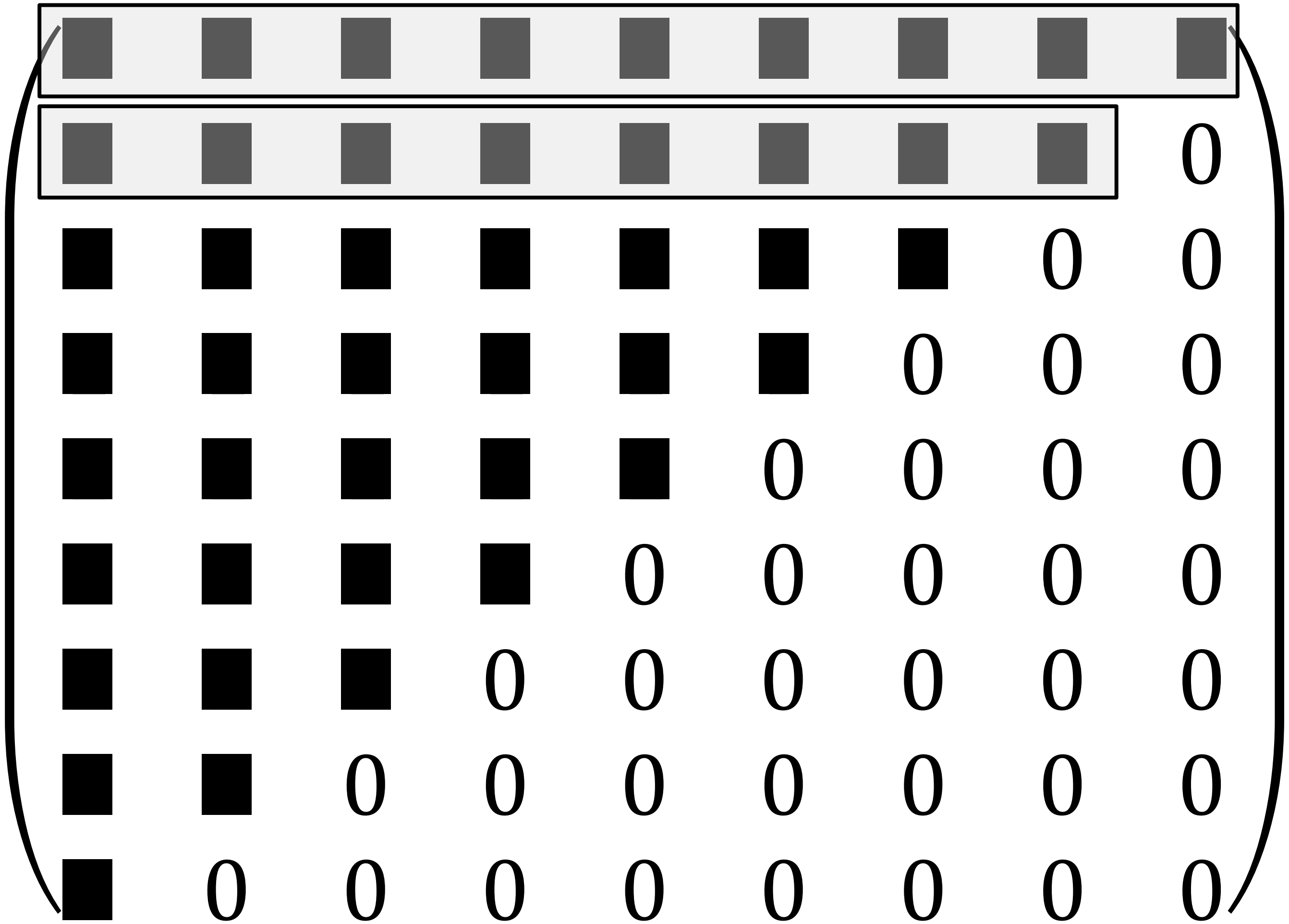
k_i = Number of interactions in row i

k_j = Number of interactions in row j

$$o_{ij} = \frac{8}{8} = 1$$

How can we quantify this pattern?

Measuring overlap between pairs of plants (rows)



c_{ij} = Number of shared interactions between row *i* and *j*

k_i = Number of interactions in row *i*

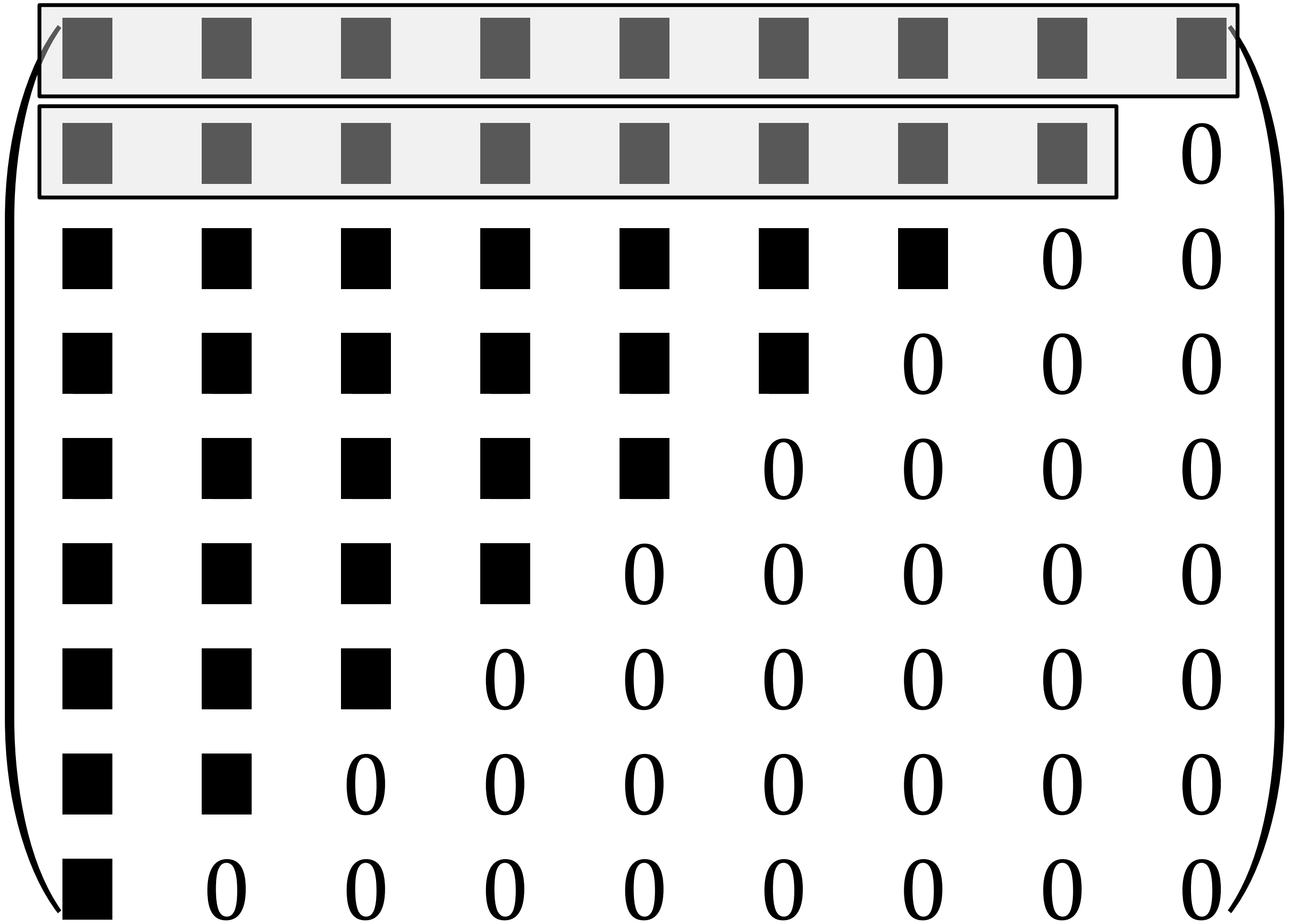
k_j = Number of interactions in row *j*

$$o_{ij} = \frac{c_{ij}}{\min(k_i, k_j)}$$

$$\sum_{i<j}^{NP} o_{ij}$$

How can we quantify this pattern?

Measuring overlap between **all possible** *ij* pairs of rows



c_{ij} = Number of shared interactions between row *i* and *j*

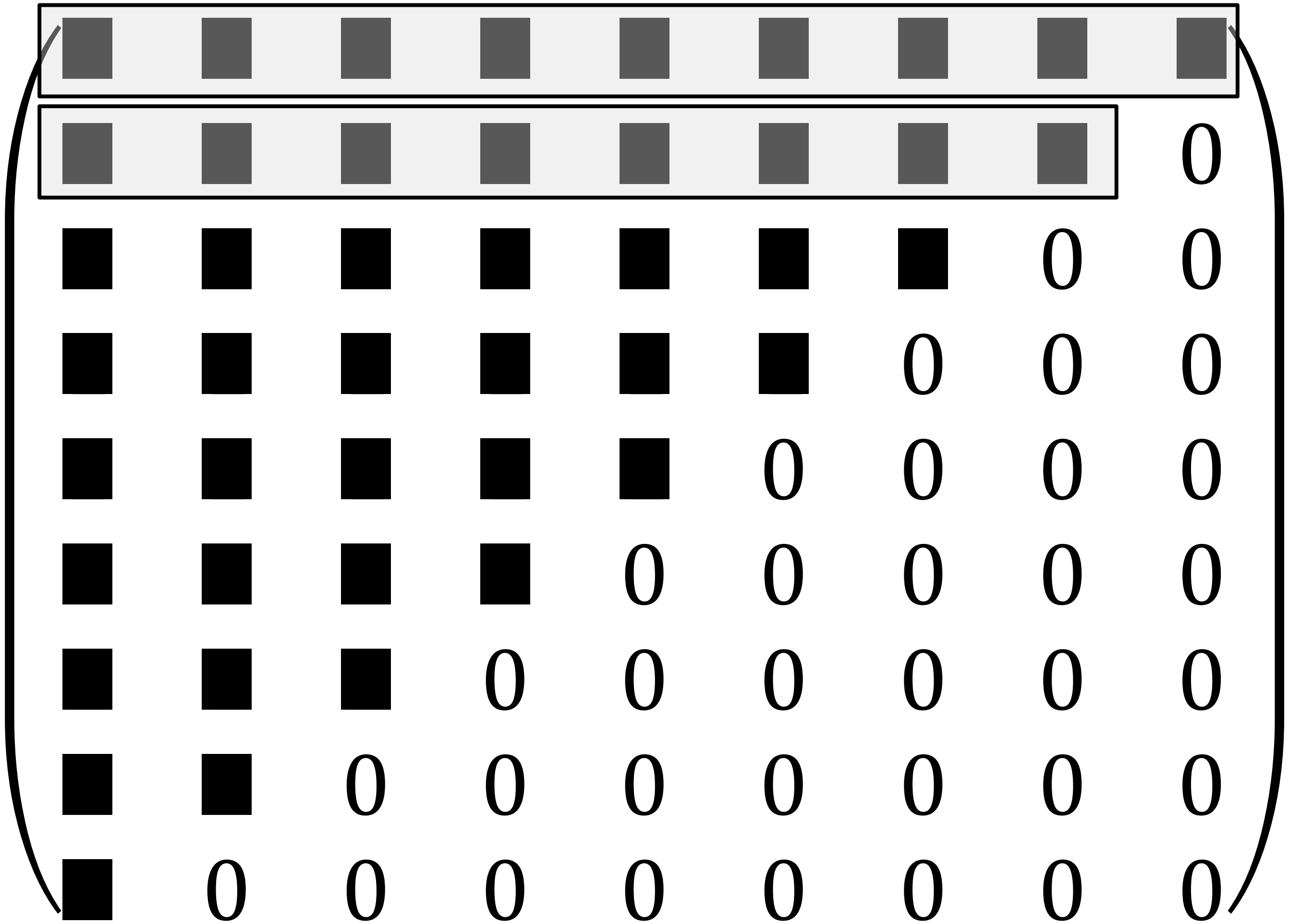
k_i = Number of interactions in row *i*

k_j = Number of interactions in row *j*

$$o_{ij} = \frac{c_{ij}}{\min(k_i, k_j)}$$

$$\frac{\sum_{i<j}^{NP} o_{ij}}{NP(NP-1)/2}$$

Measuring overlap between **all possible** *ij* pairs of rows
Divide by the number of possible pairs



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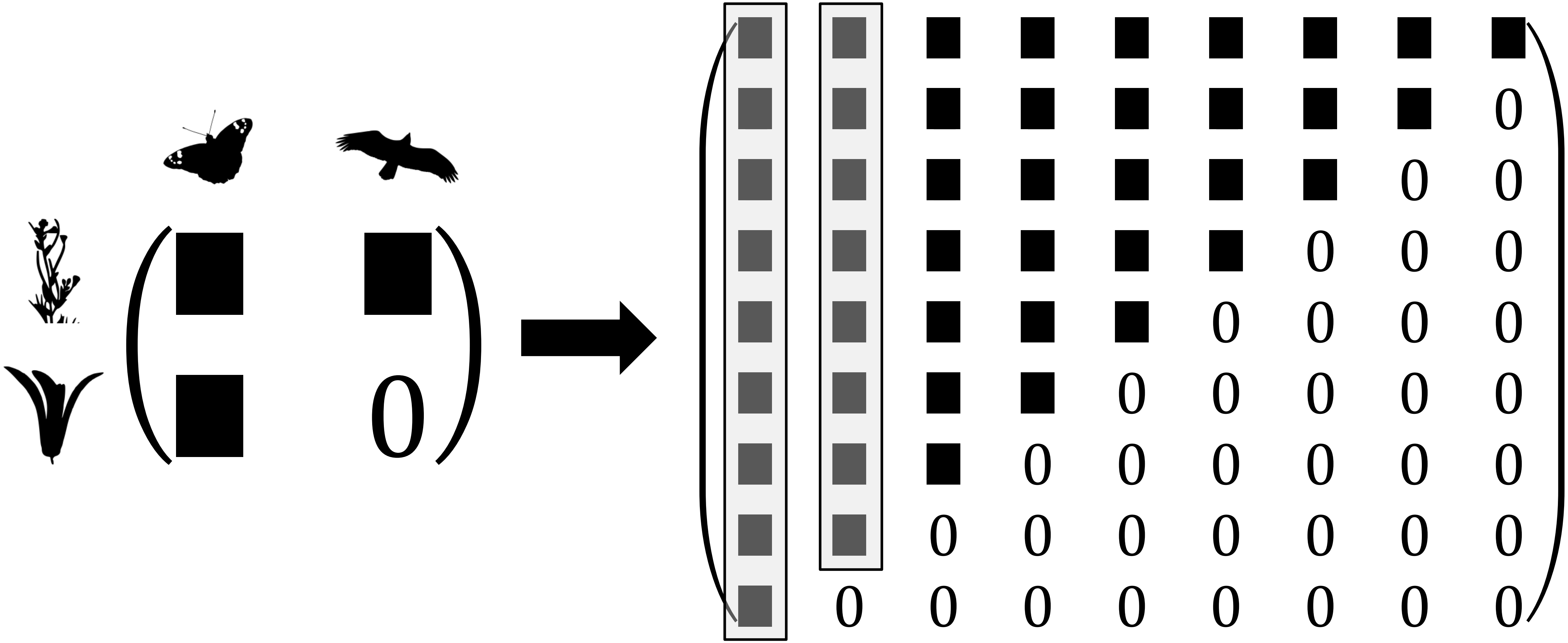
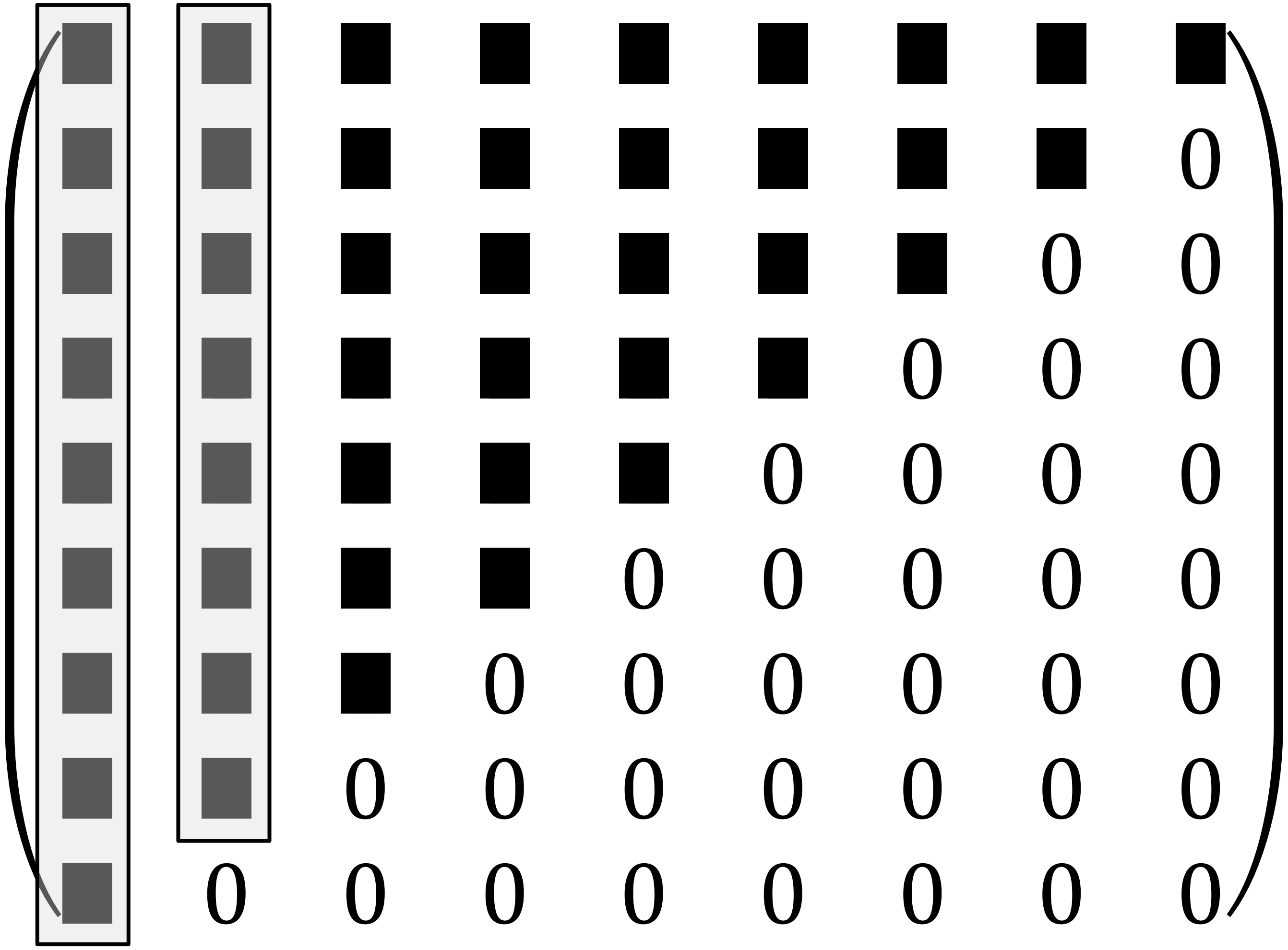
Measuring overlap between **all possible** *ij* pairs of rows
Divide by the number of possible pairs



Interactions of animals with a lower degree are contained within the interactions of animals with a higher degree



c_{ij} = Number of shared interactions between row i and j

k_i = Number of interactions in row i

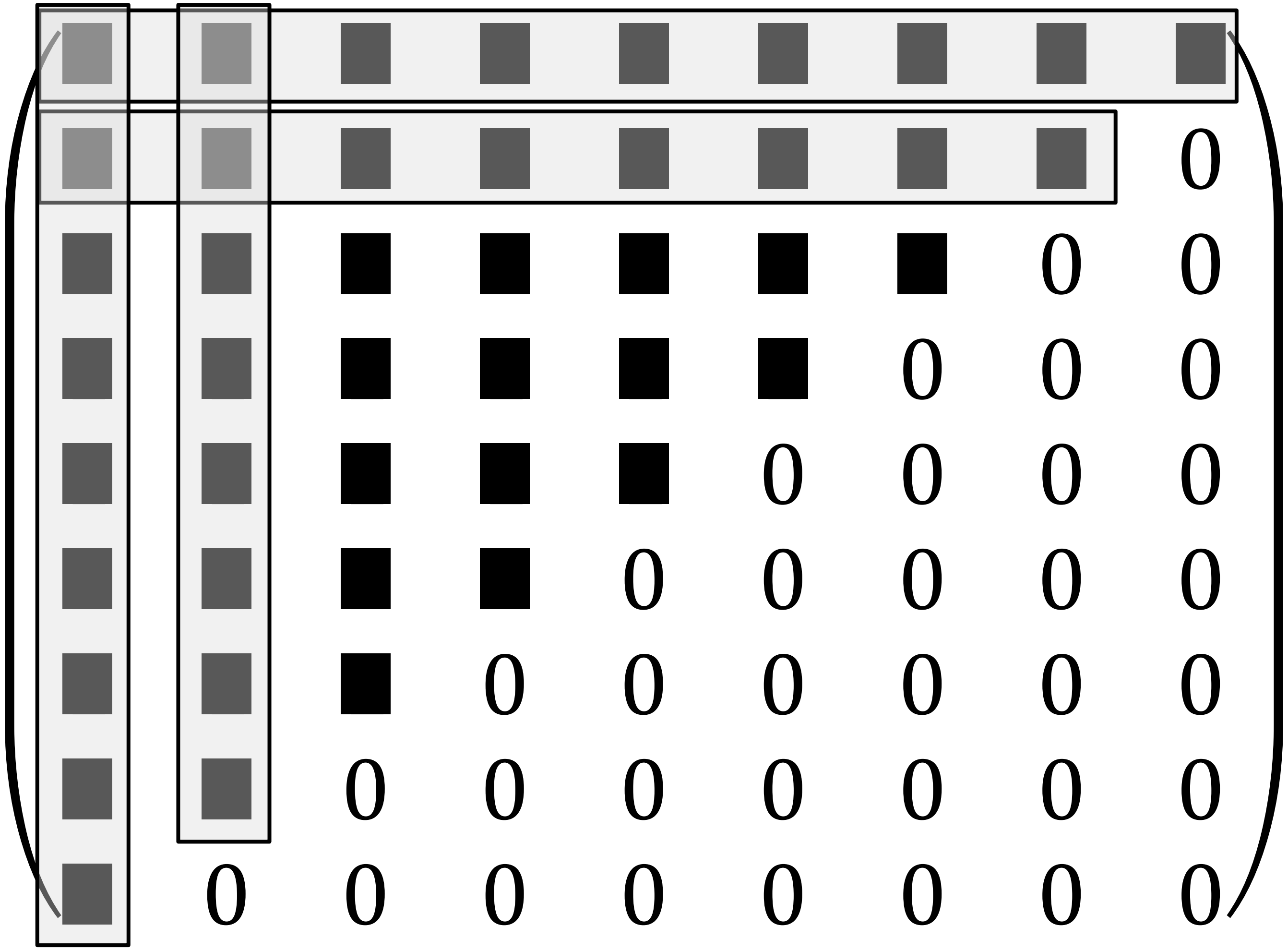
k_j = Number of interactions in row j

$$o_{ij} = \frac{c_{ij}}{\min(k_i, k_j)}$$

$$\frac{\sum_{i<j}^{NA} o_{ij}}{NA(NA-1)/2}$$

Repeat for the columns:

Measuring overlap between **all possible** *ij* pairs of columns



c_{ij} = Number of shared interactions between row i and j

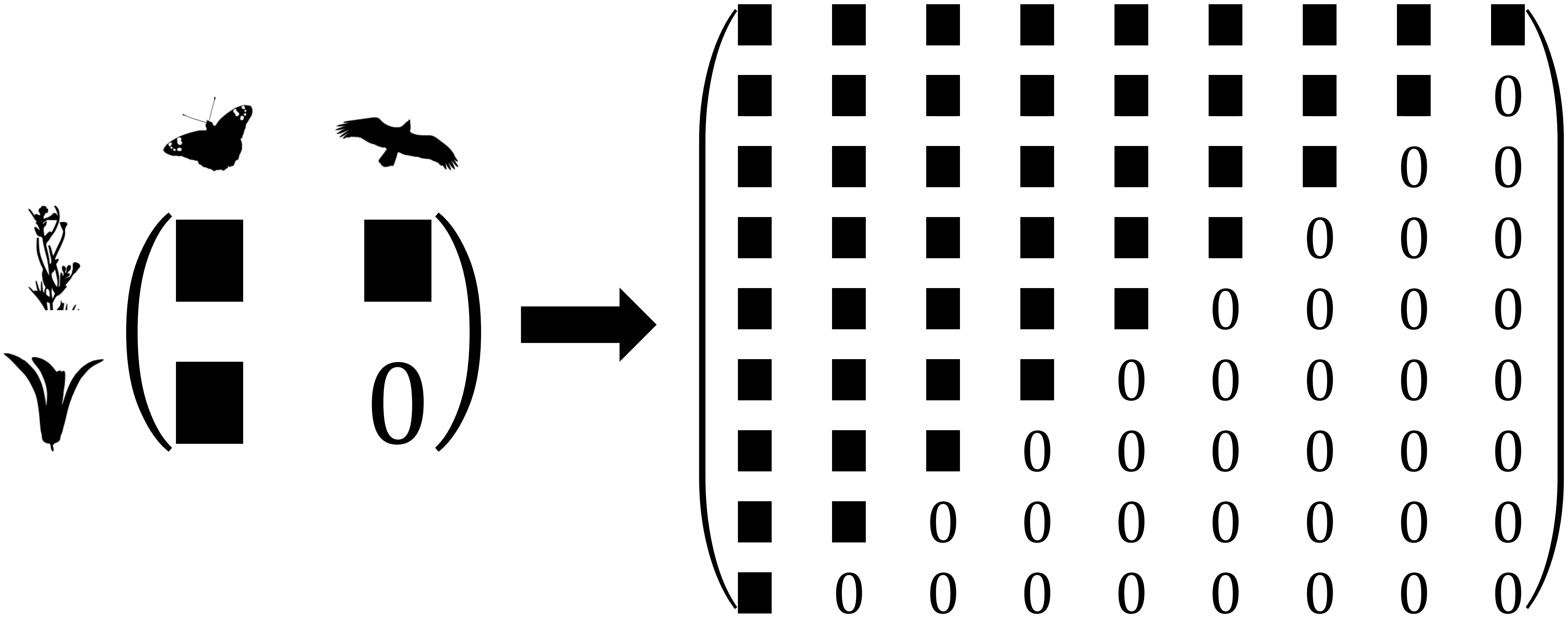
k_i = Number of interactions in row i

k_j = Number of interactions in row j

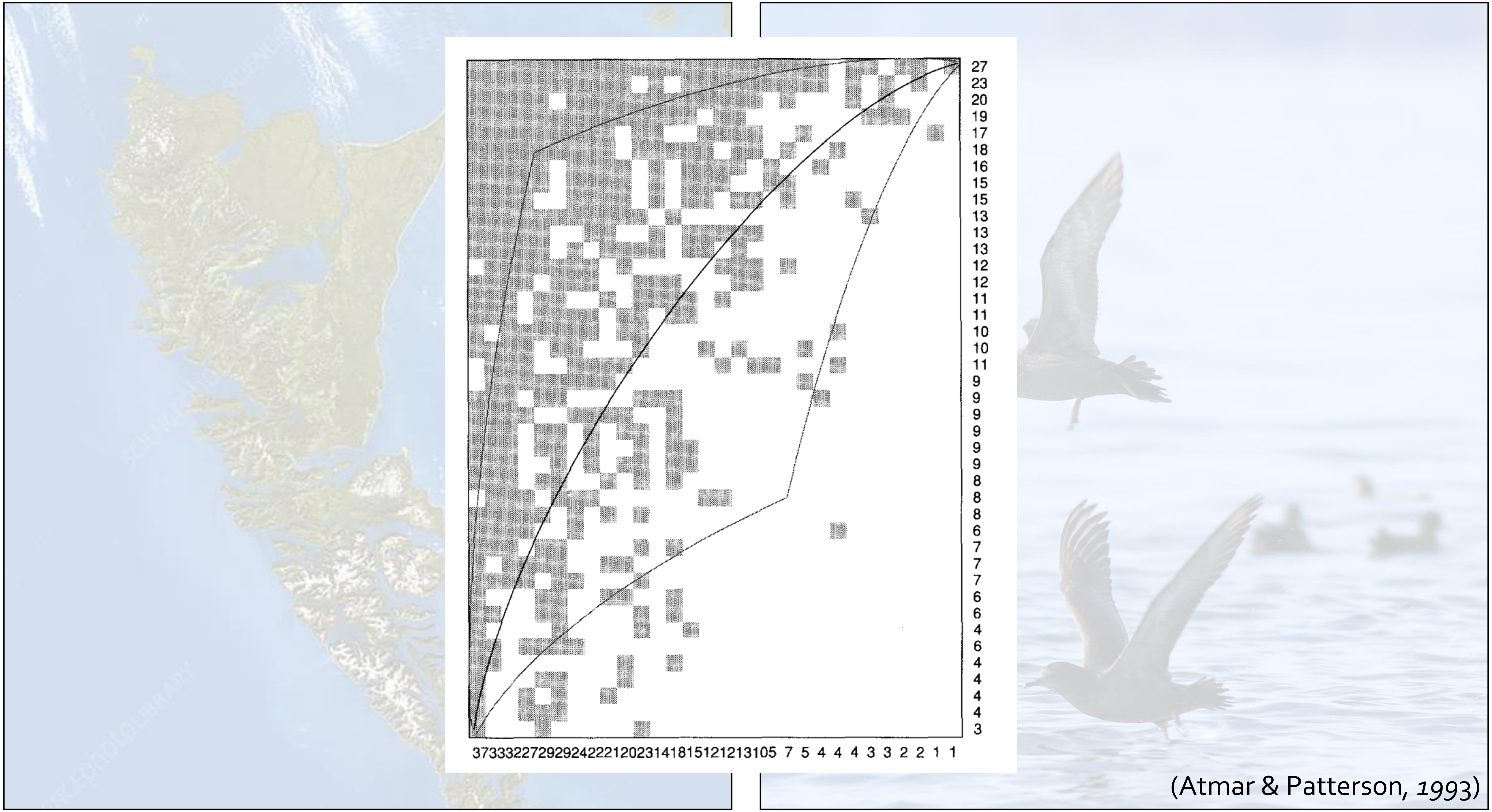
$$\eta = \frac{\sum_{i<j}^{NP} o_{ij} + \sum_{i<j}^{NA} o_{ij}}{\frac{NP(NP-1)}{2} + \frac{NA(NA-1)}{2}}$$

A general formula for quantifying nestedness:

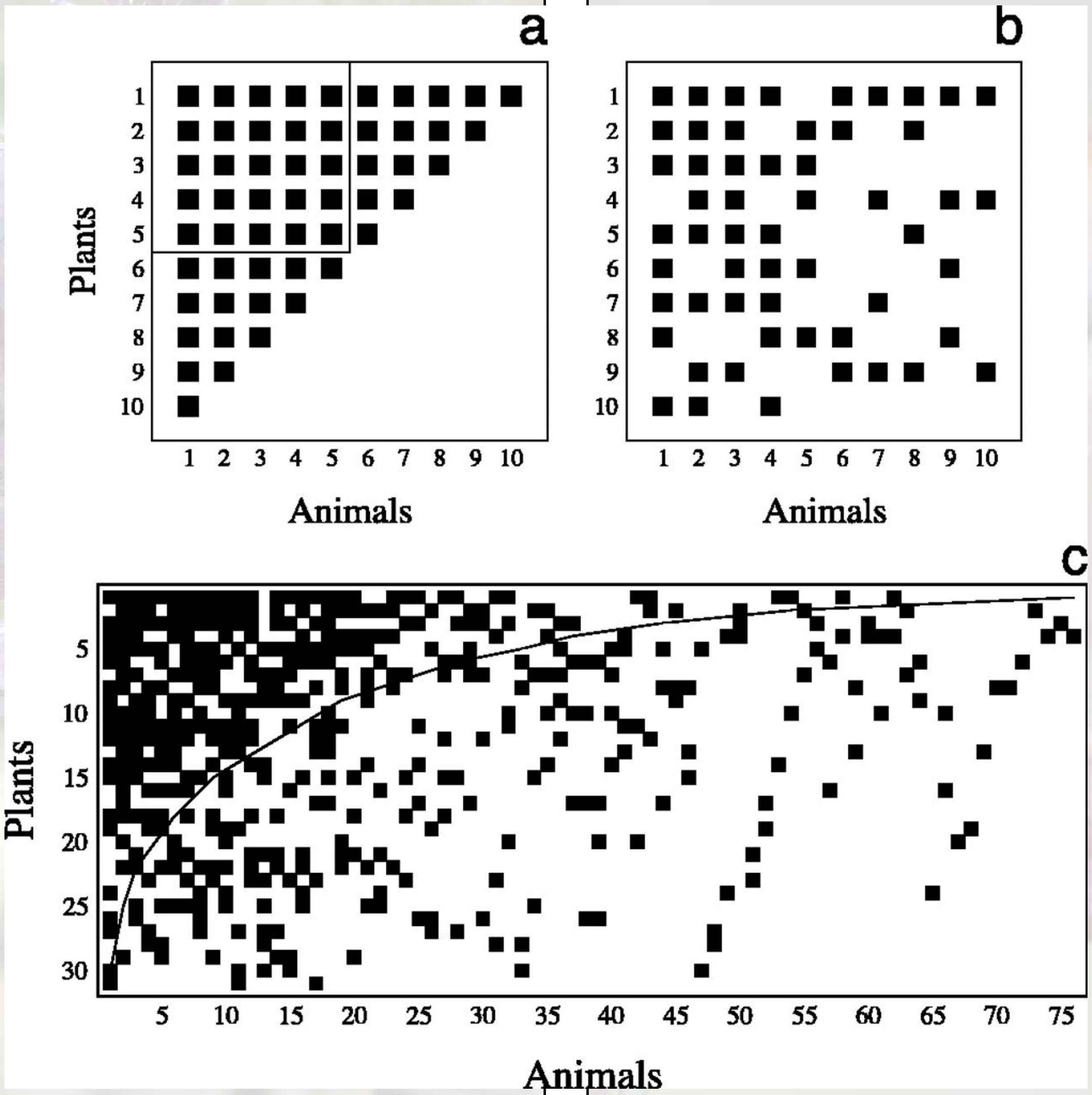
η varies between 0 and 1



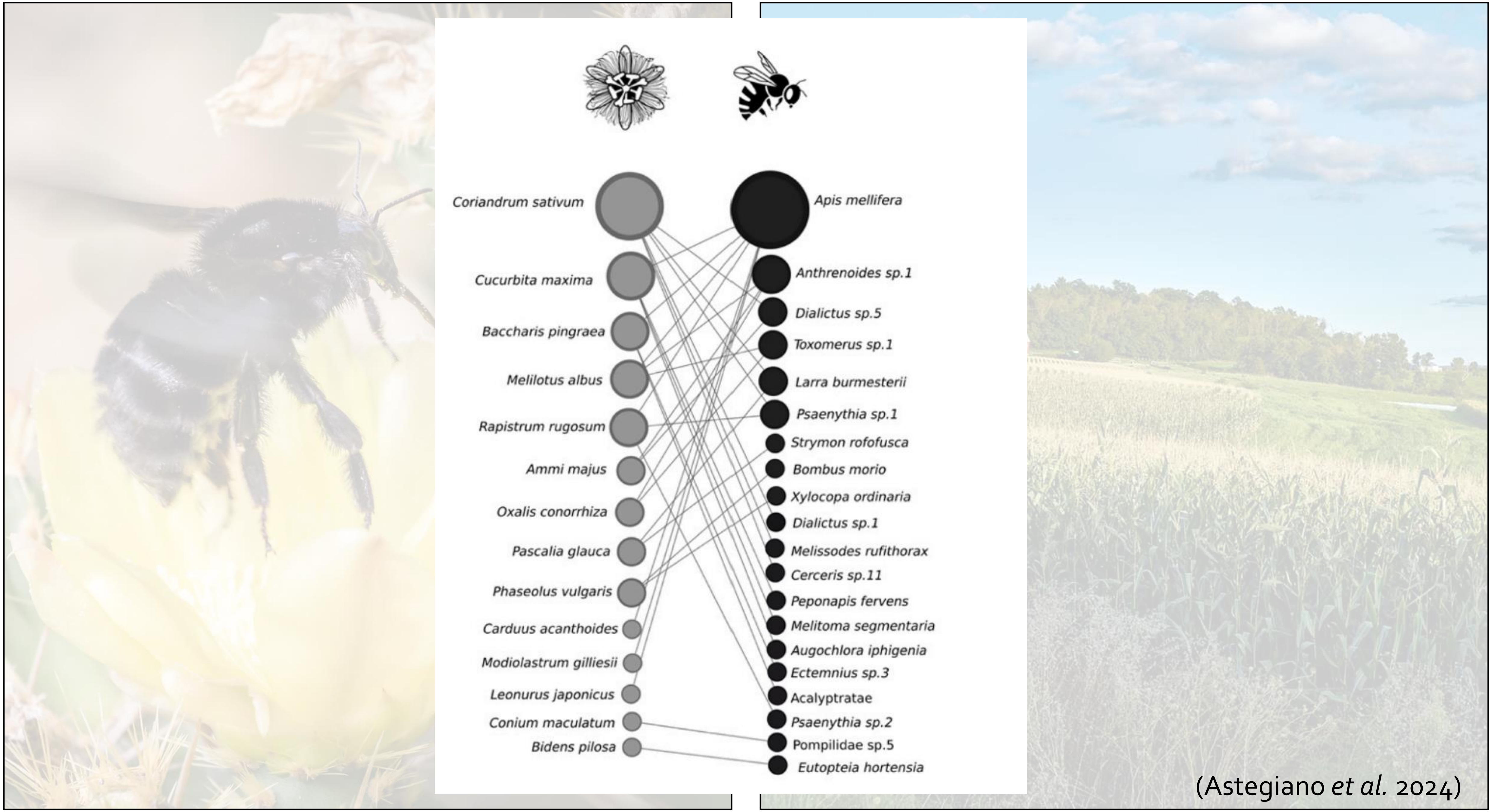
Now that we now how to quantify a nested pattern:
Where can we find it in nature?



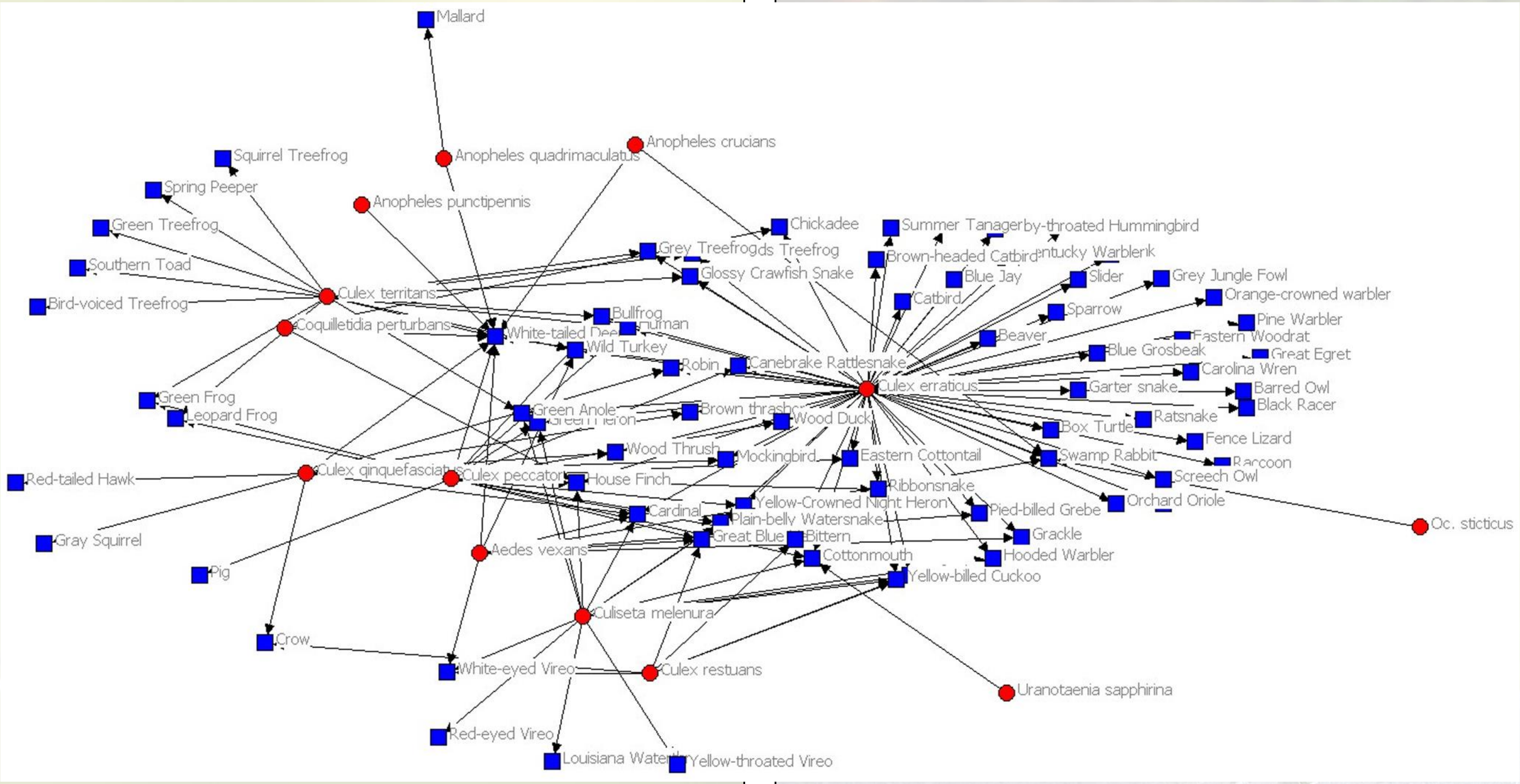
(Atmar & Patterson, *1993*)

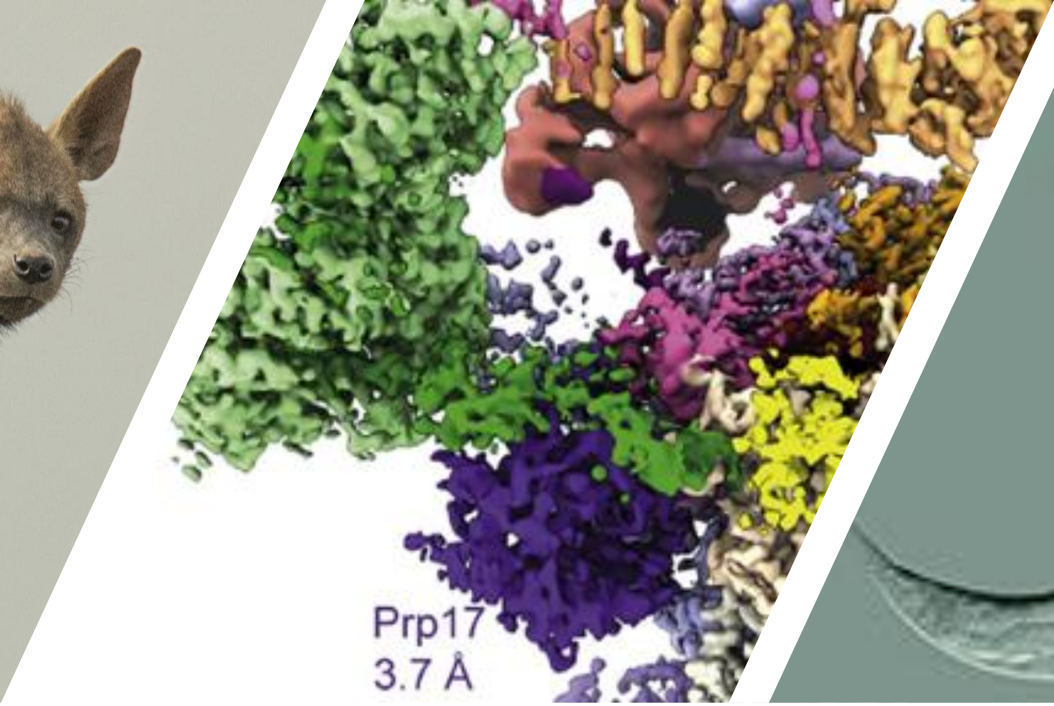
(Bascompte *et al.* 2003)

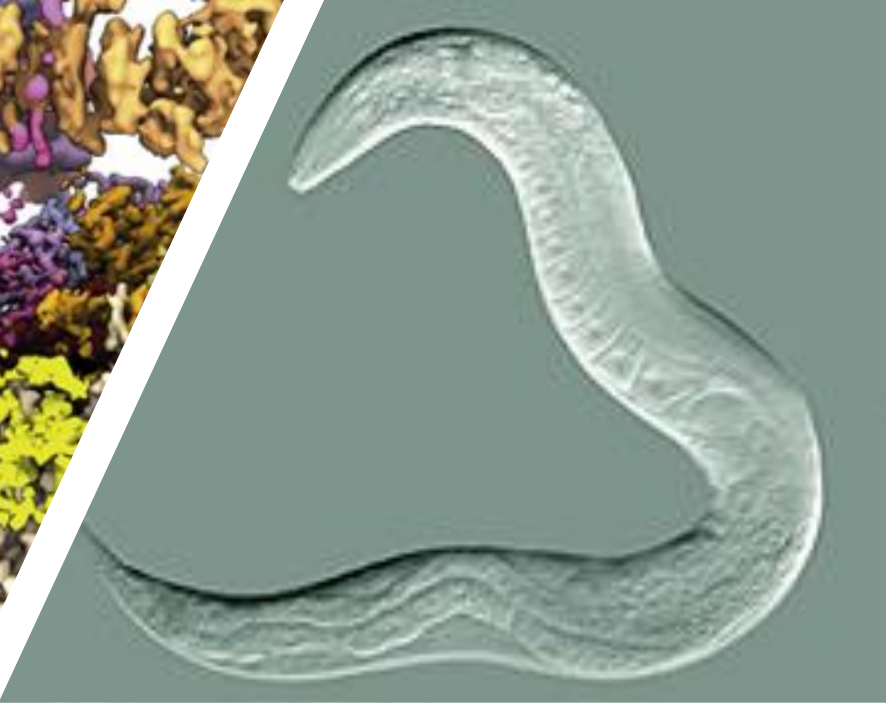


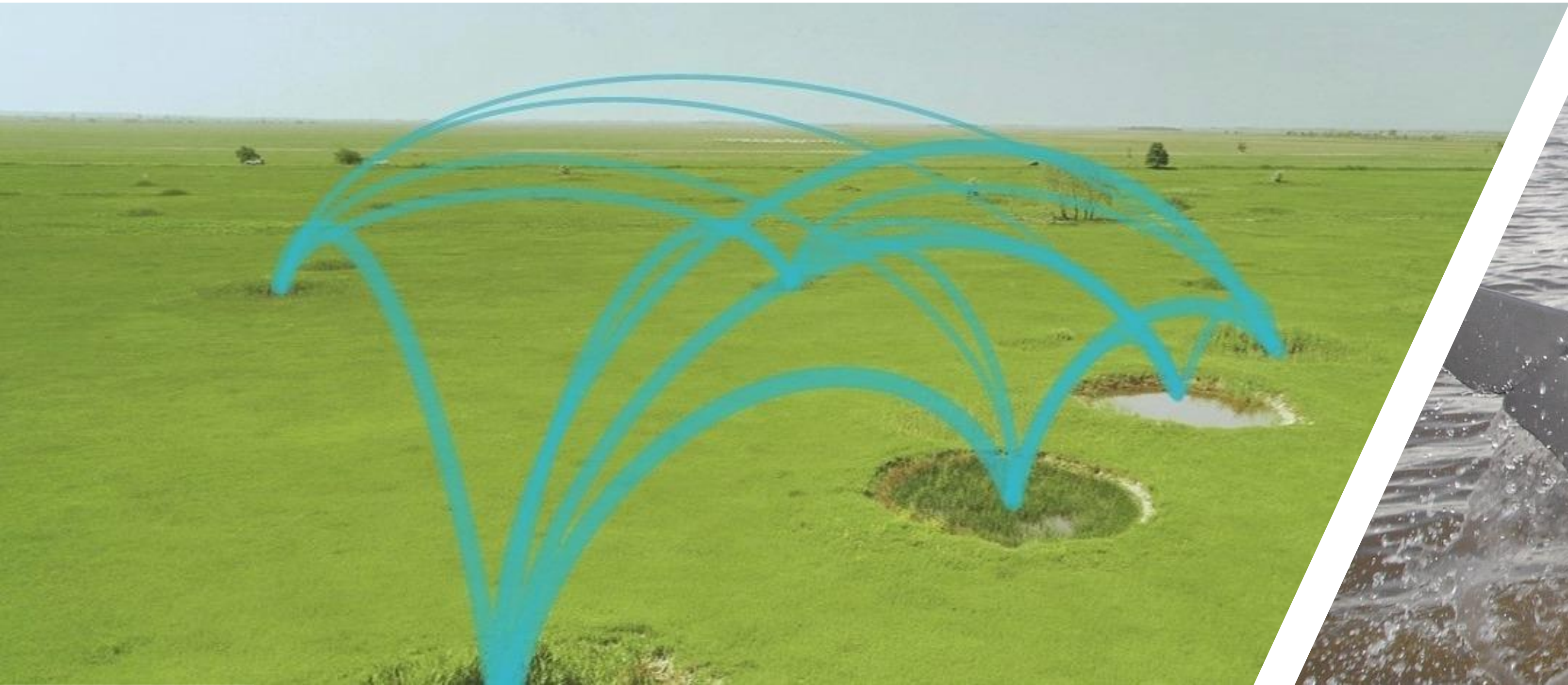
(Astegiano *et al.* 2024)

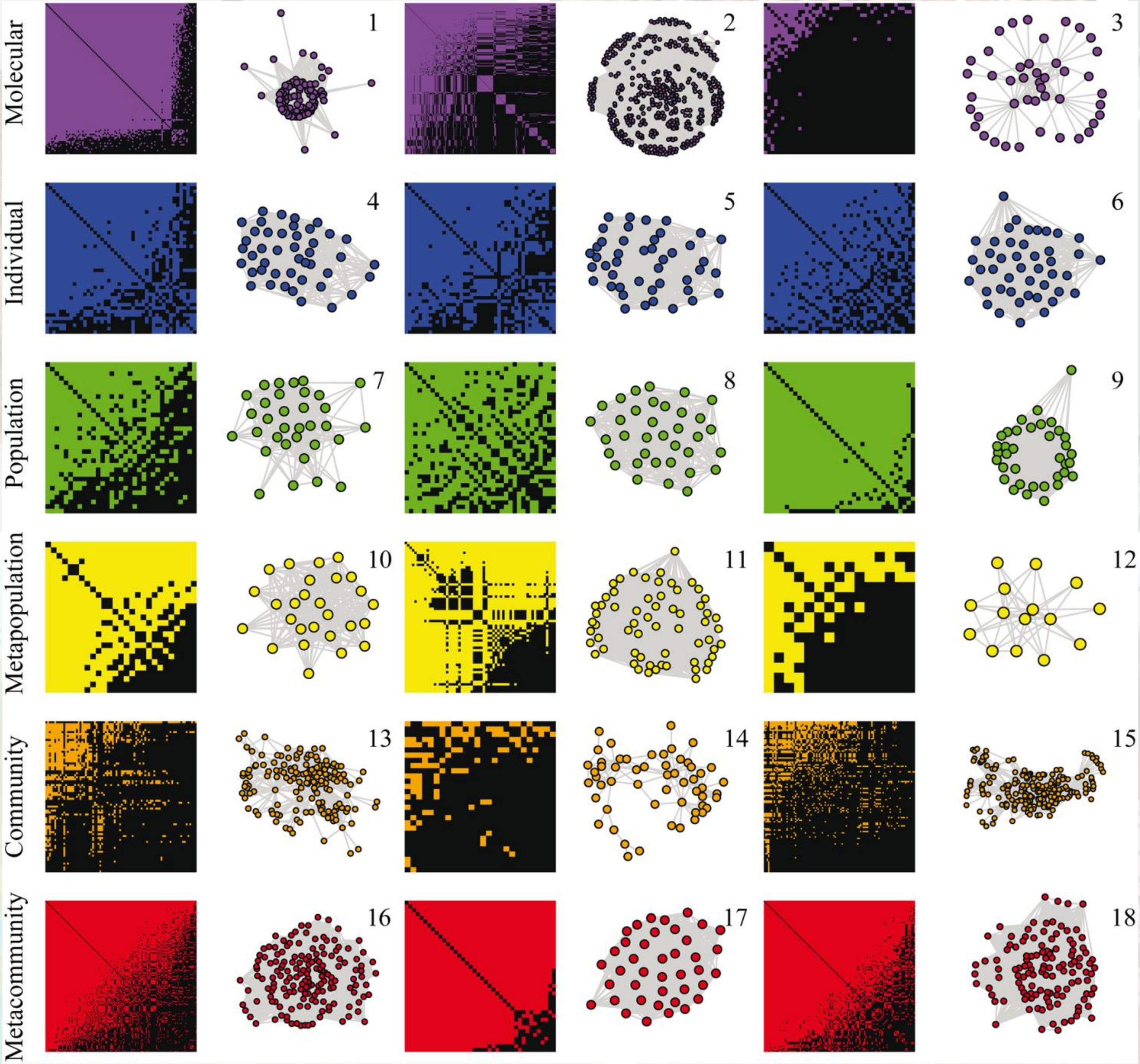



(Graham *et al.* 2009)

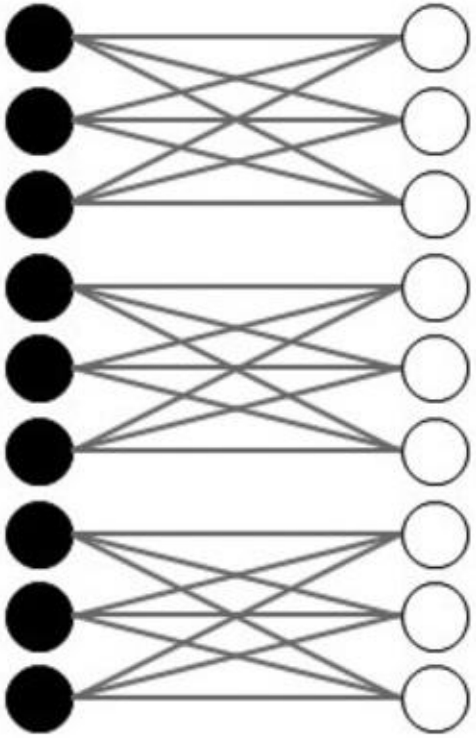

Prp17
3.7 Å





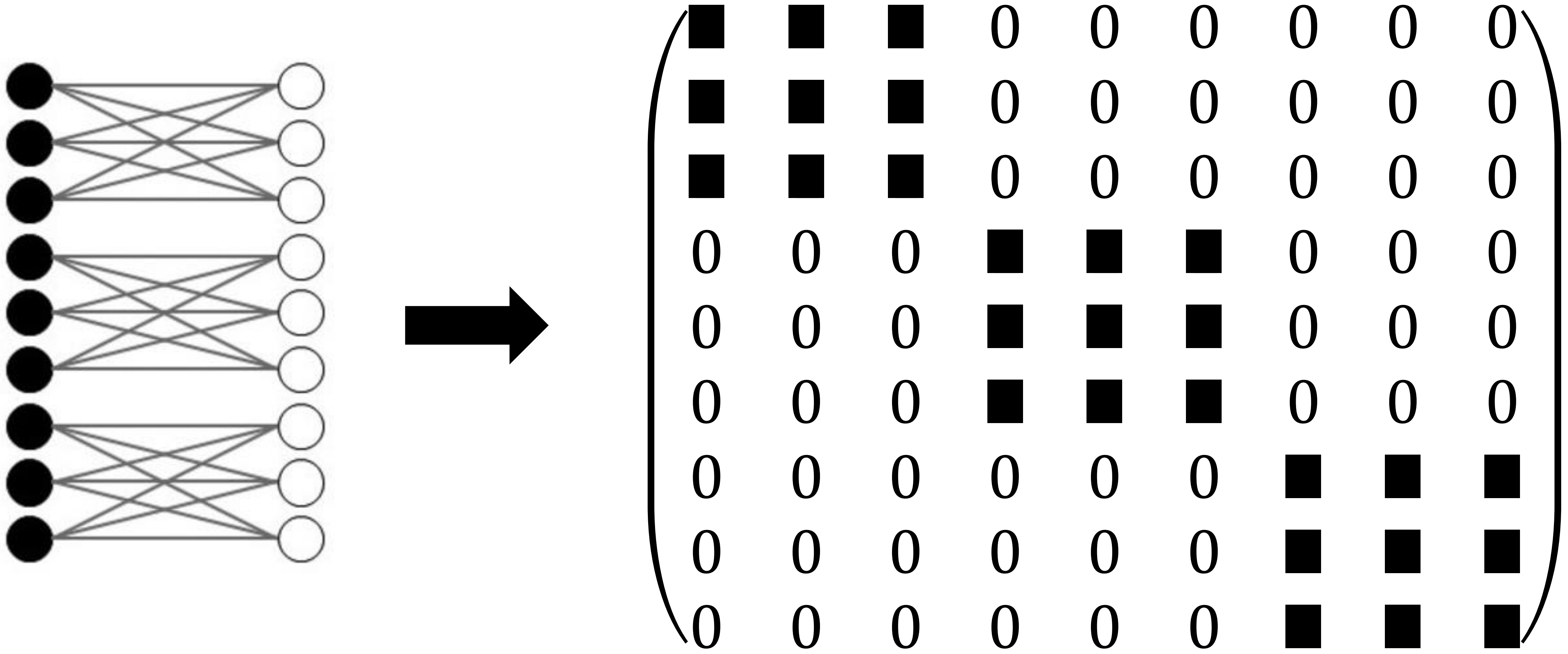


(Cantor *et al.* 2017)

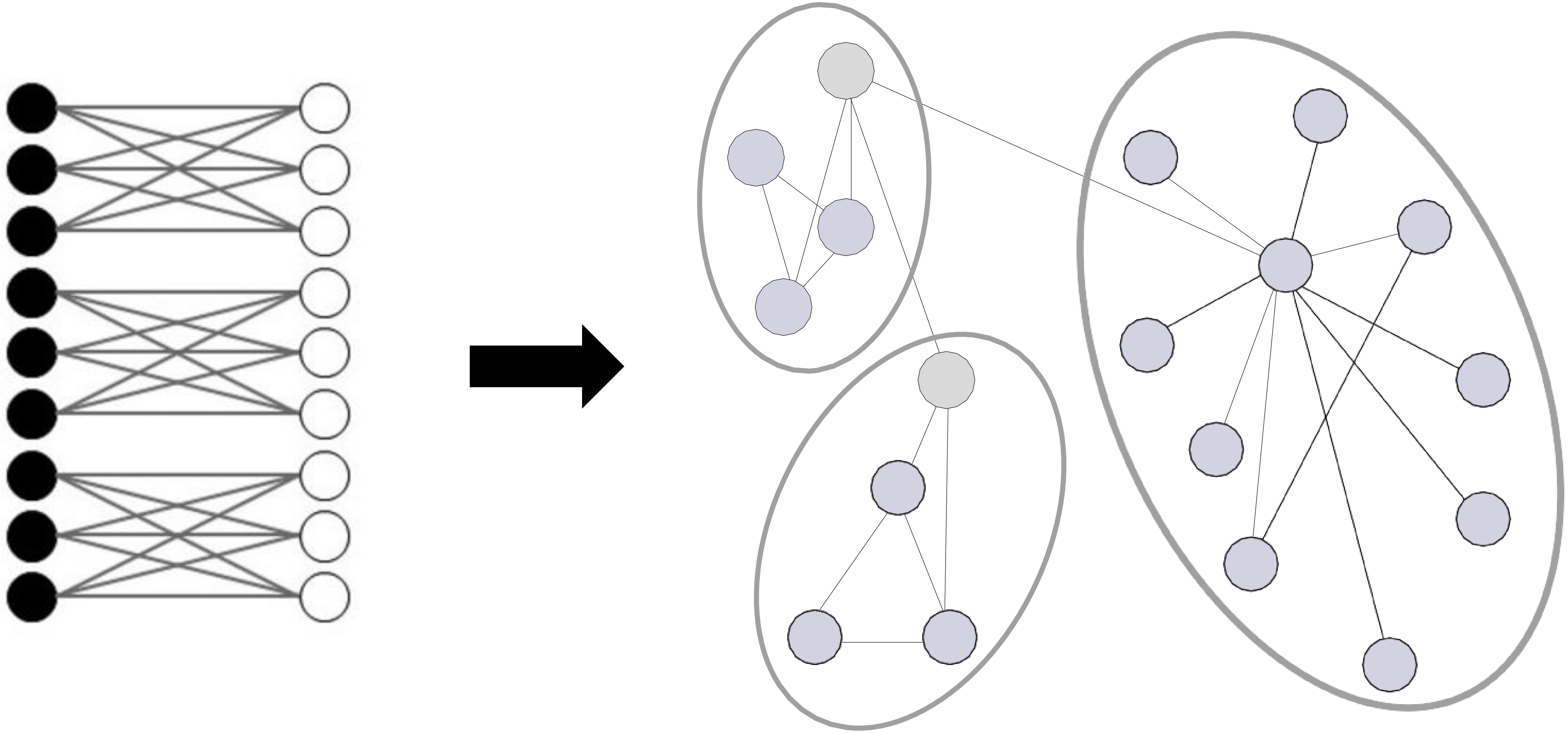


■	■	■	0	0	0	0	0	0
■	■	■	0	0	0	0	0	0
■	■	■	0	0	0	0	0	0
0	0	0	■	■	■	0	0	0
0	0	0	■	■	■	0	0	0
0	0	0	■	■	■	0	0	0
0	0	0	0	0	0	■	■	■
0	0	0	0	0	0	■	■	■
0	0	0	0	0	0	■	■	■

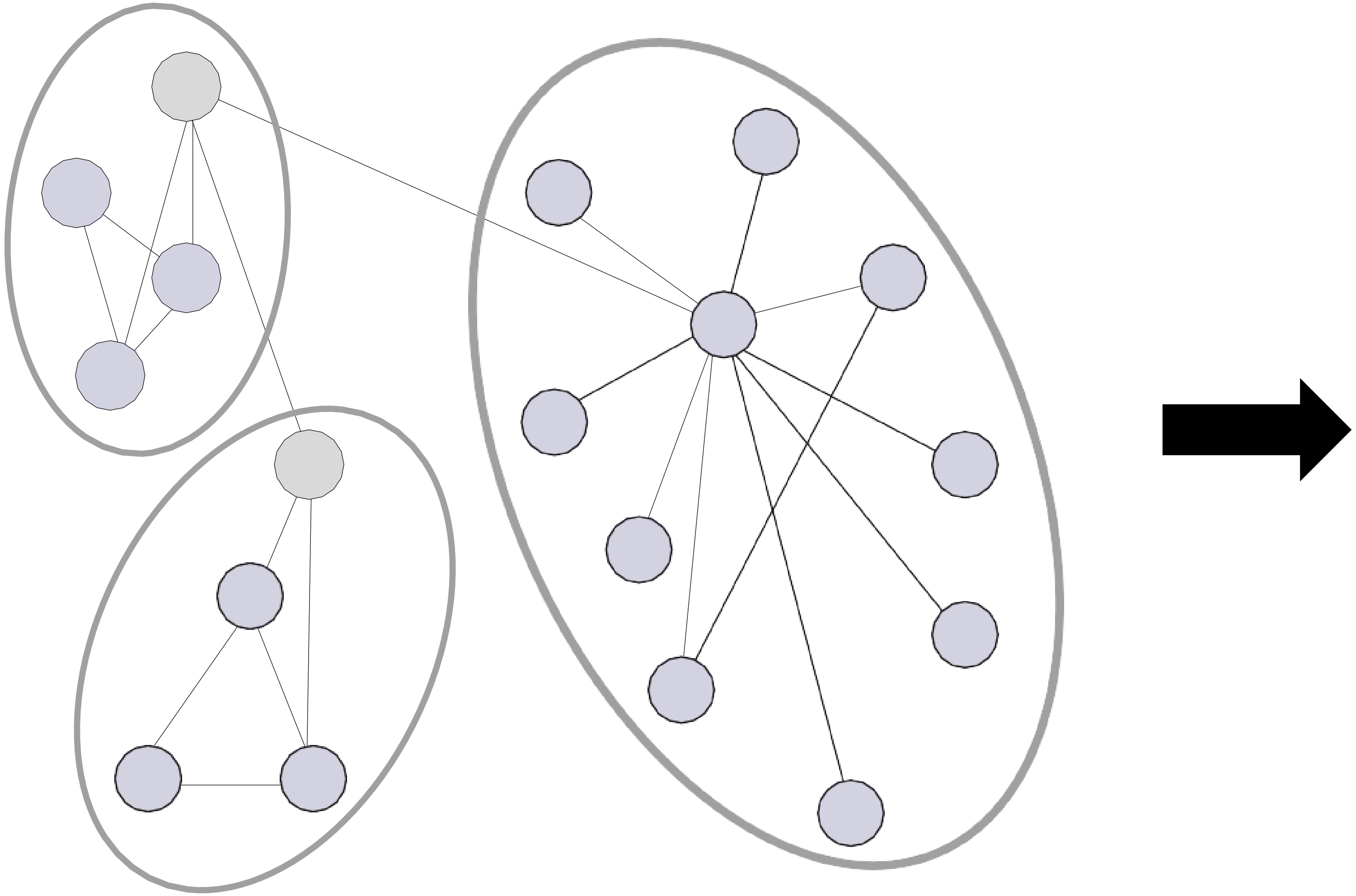
What about networks that look like this?
Division into subgroups or "modules"



Division into subgroups or "modules"
Extreme case: completely isolated subnetworks

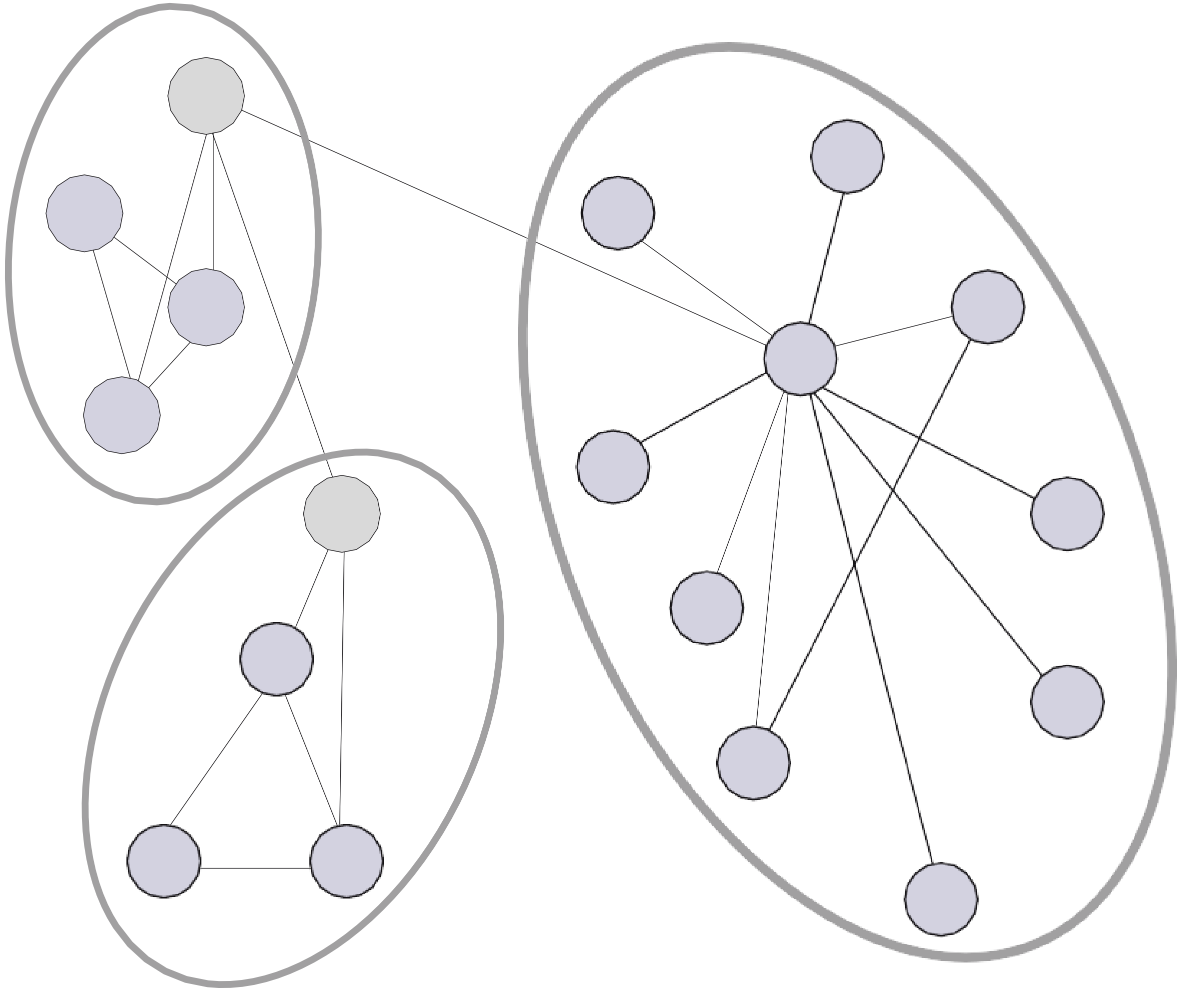


Modularity: species interact much more frequently within the same module than they do with species from other modules



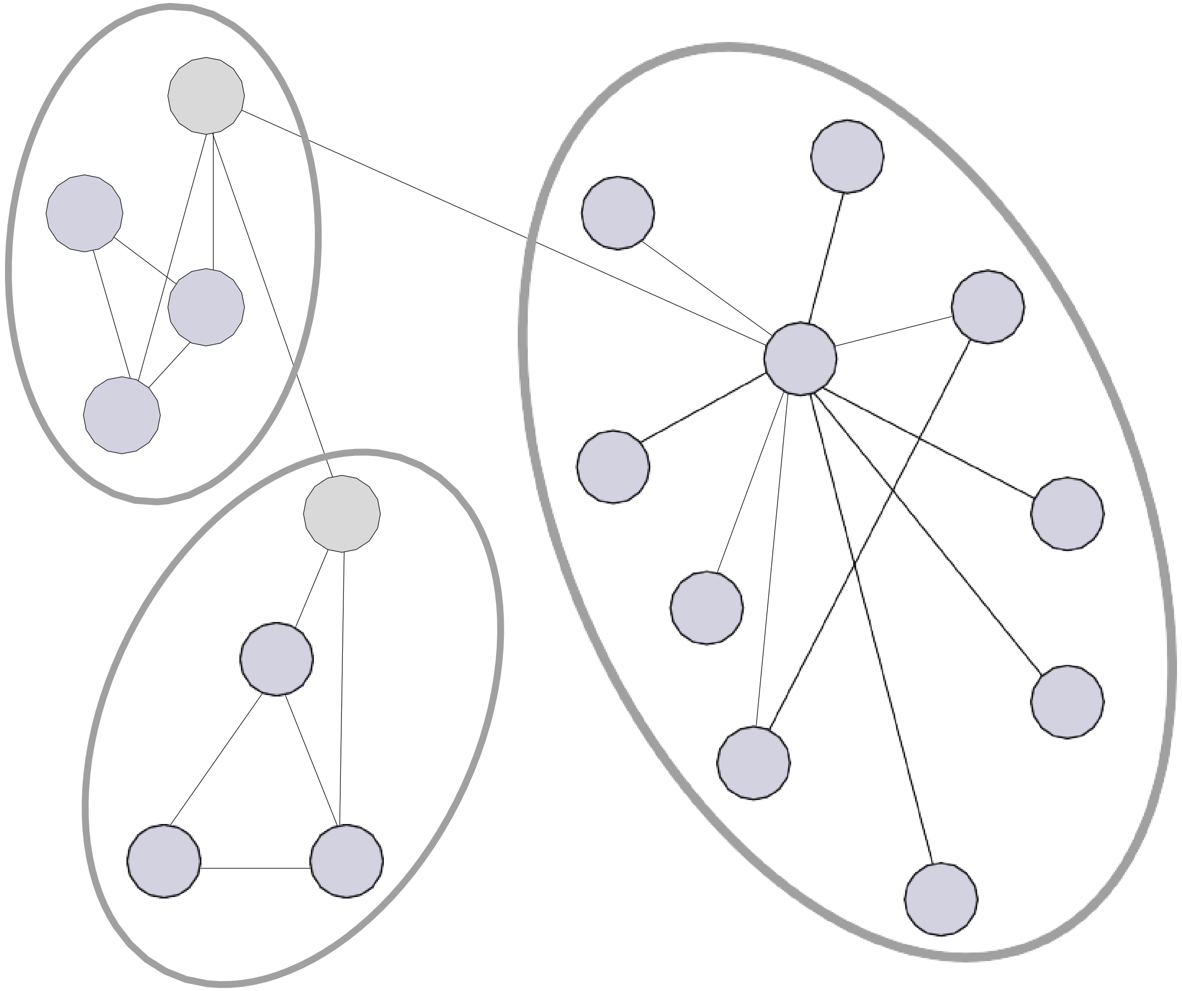
Fraction of interactions within modules vs among modules

How can we measure modularity?



Fraction of interactions within modules vs among modules

How to find the modules? What is the optimal way to divide the network?



$l_s =$ Number of interactions within module s

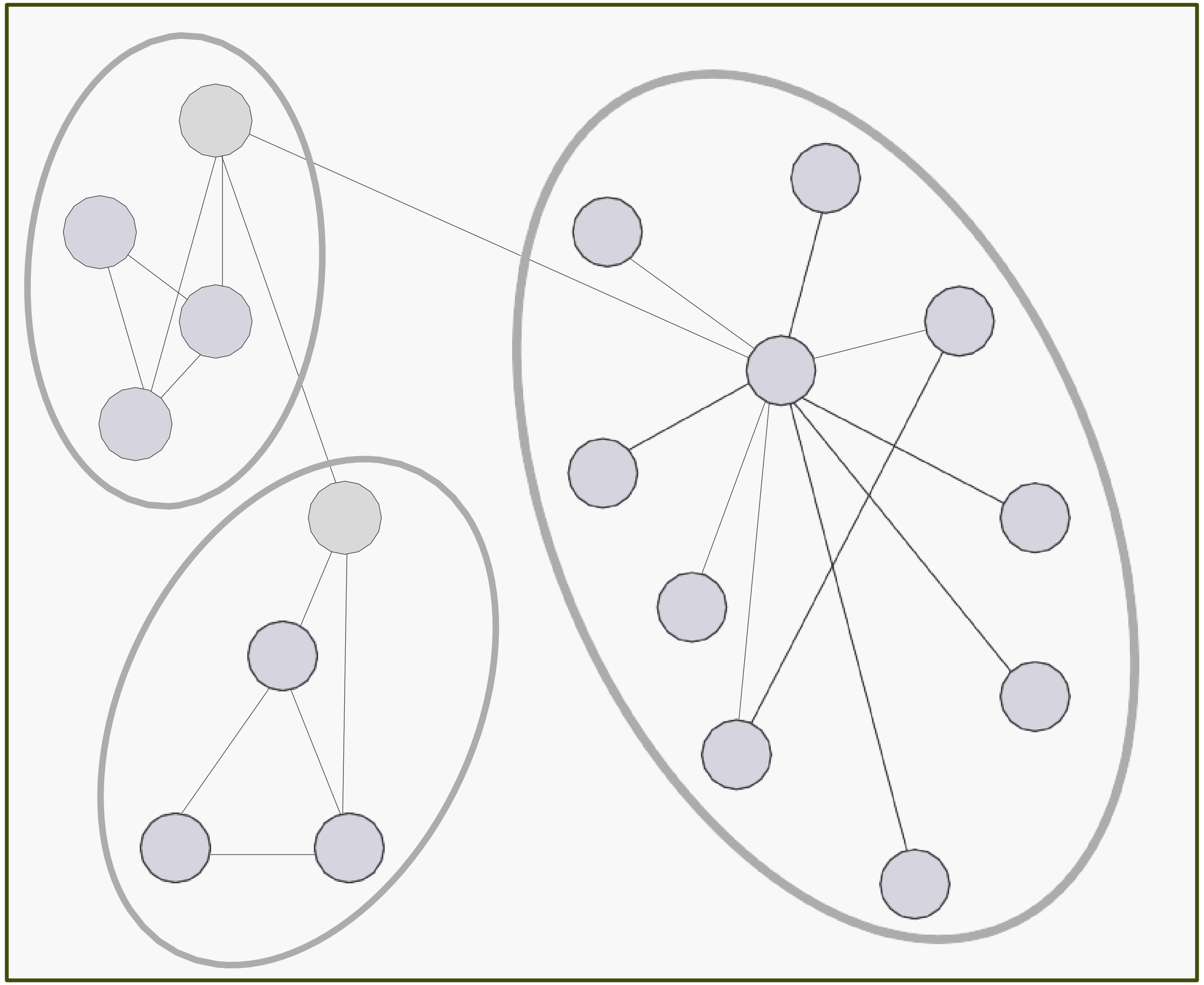
$L =$ Number of interactions in the network

Maximize $\frac{l_s}{L}$

For all modules s

How to find the modules?

Dividing the network to maximize within modules interactions



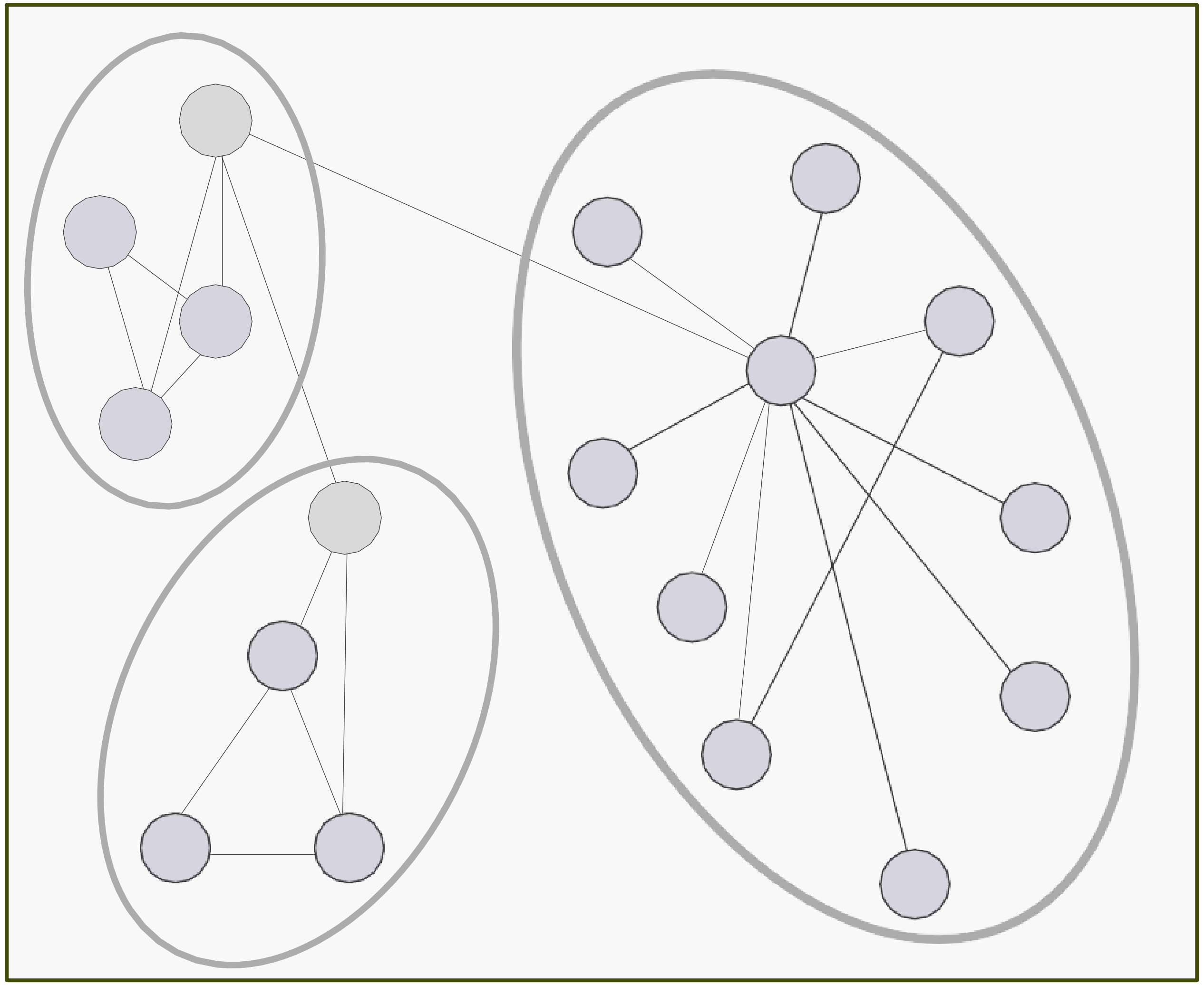
l_s = Number of interactions within module s

L = Number of interactions in the network

Maximize $\frac{l_s}{L}$

For all modules *s*

The problem: we may end up with trivial ”optimal divisions”



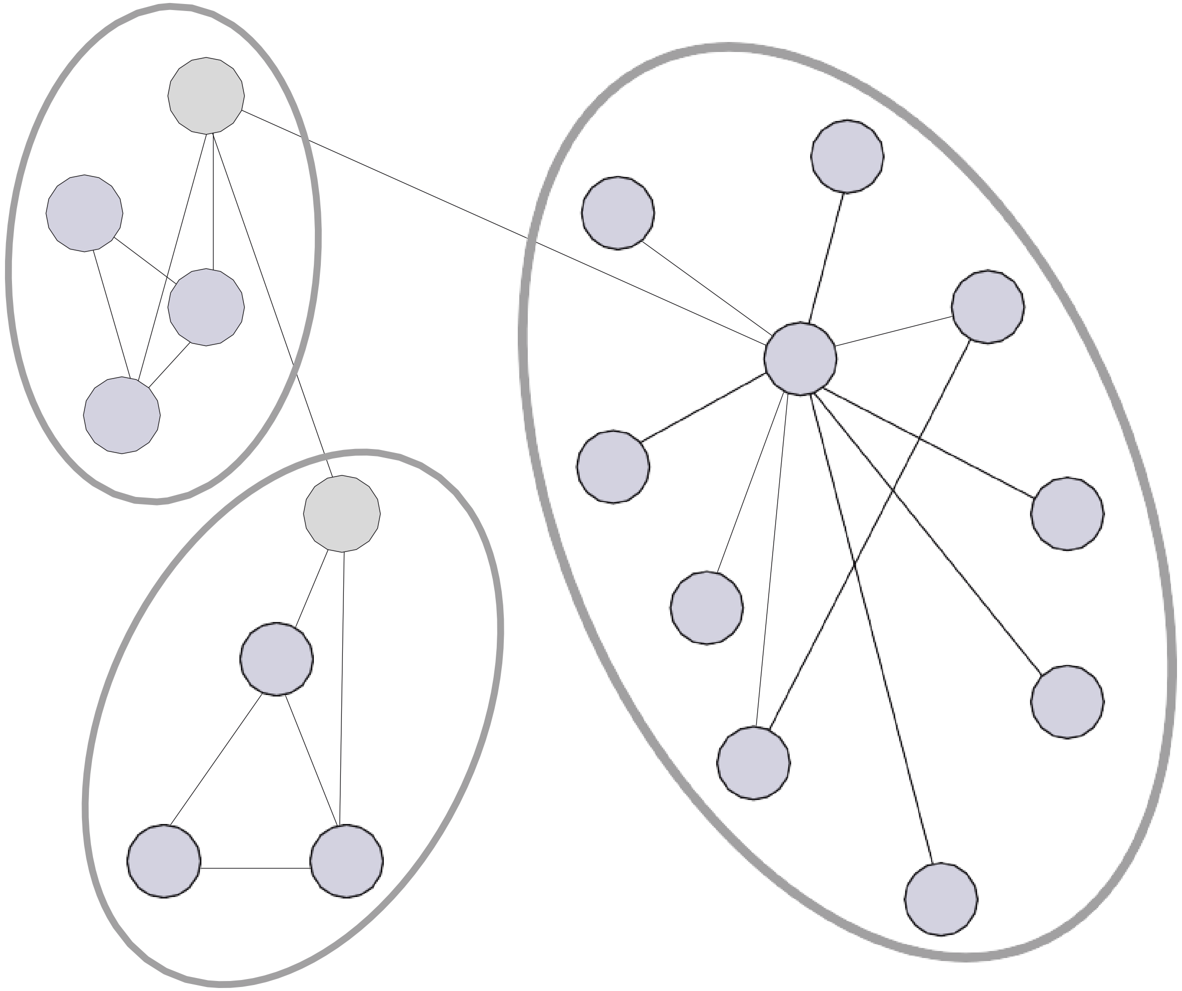
l_s = Number of interactions within module s

L = Number of interactions in the network

Maximize $\frac{l_s}{L}$

For all modules s

The solution: maximize within modules interactions relative to a randomized version of the same network



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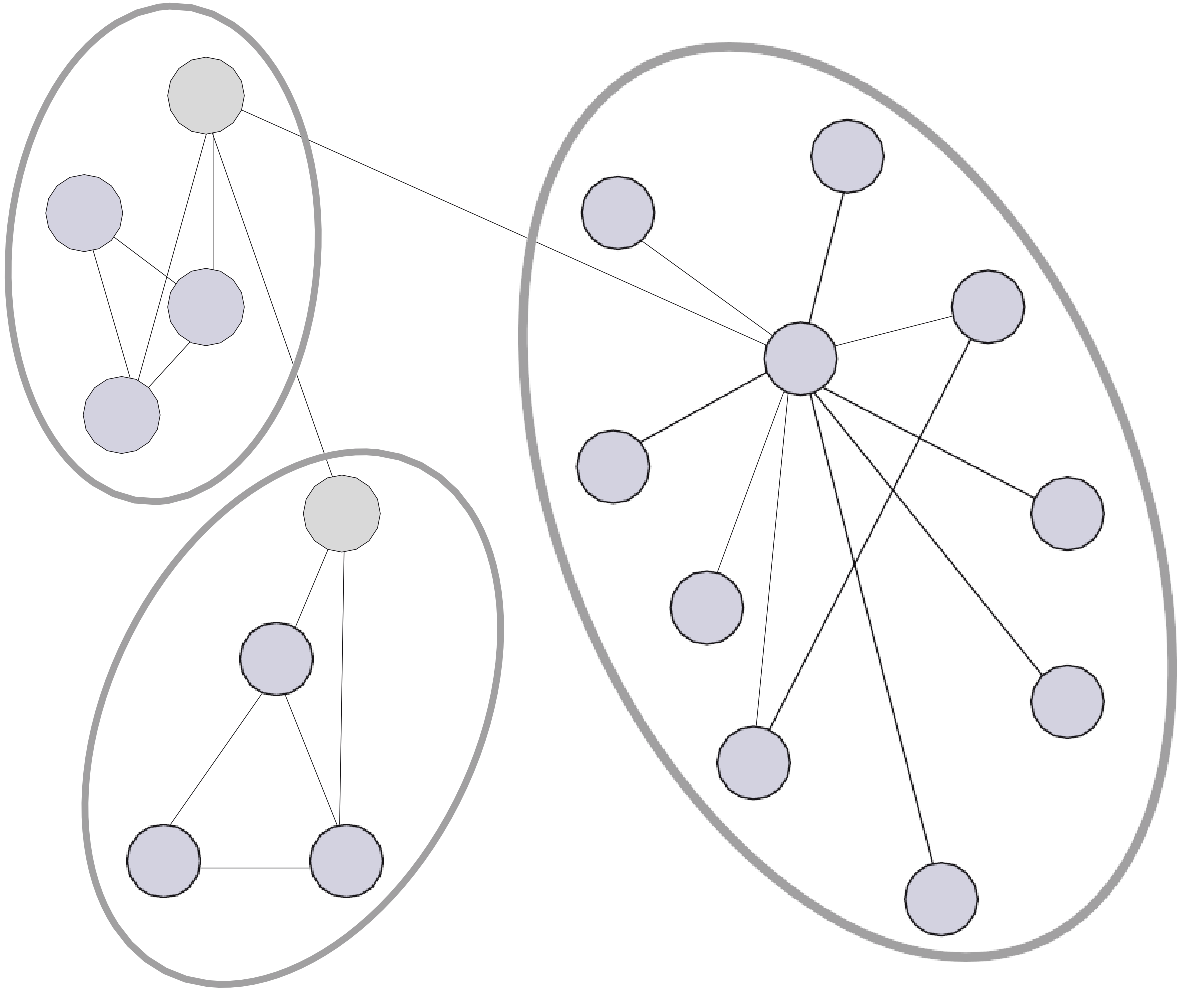


l_s = Number of interactions within module s

L = Number of interactions in the network

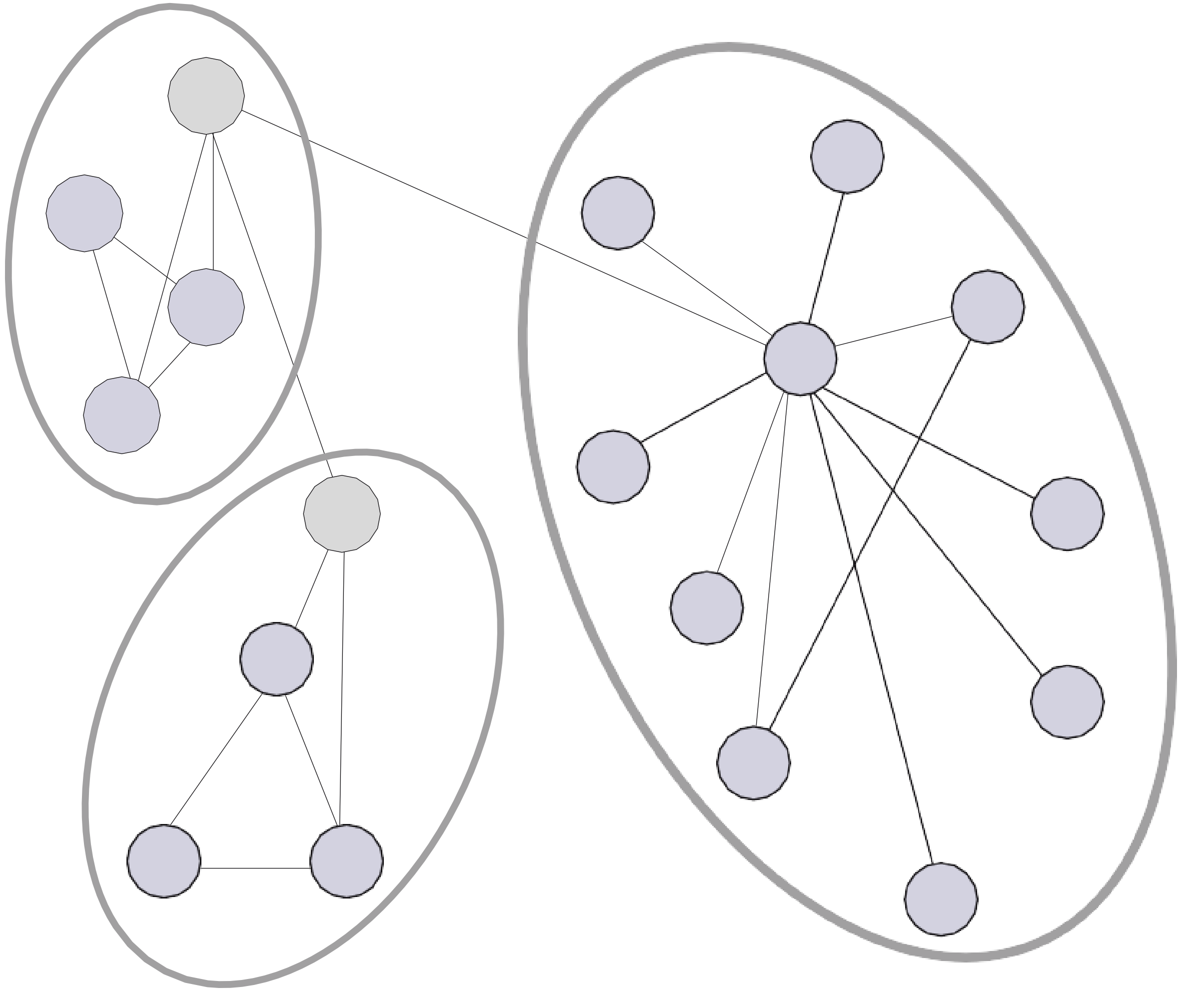
Maximize

$$\frac{l_s}{L} - \left(\frac{d_s}{L}\right)^2$$

For all modules s

d_s = Sum of the degrees of species in module s

The solution: maximize within modules interactions relative to a randomized version of the same network



l_s = Number of interactions within module s

L = Number of interactions in the network

$$M = \sum_{s=1}^{N_m} \left[\frac{l_s}{L} - \left(\frac{d_s}{L} \right)^2 \right]$$

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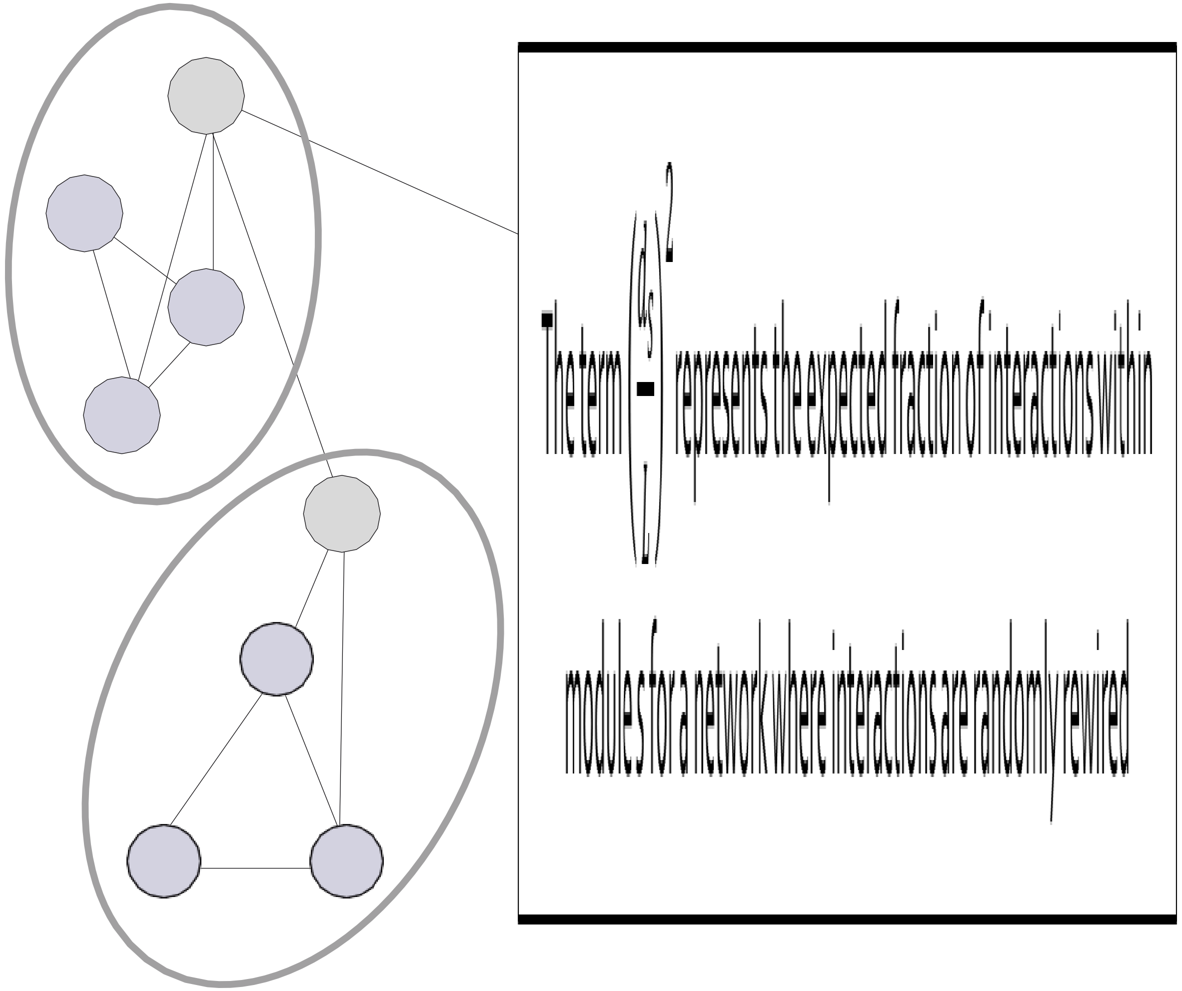
The solution: maximize within modules interactions relative to a randomized version of the same network



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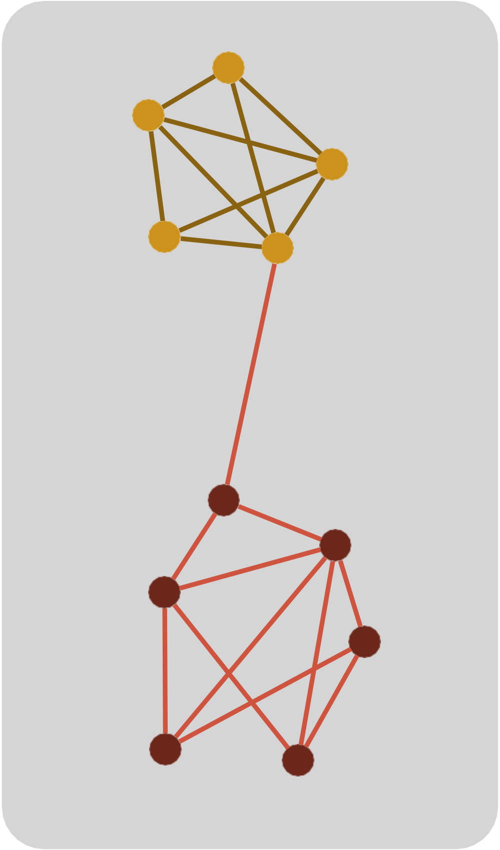
$$M = \sum_{s=1}^{N_m} \left[\frac{l_s}{L} - \left(\frac{d_s}{L} \right)^2 \right]$$

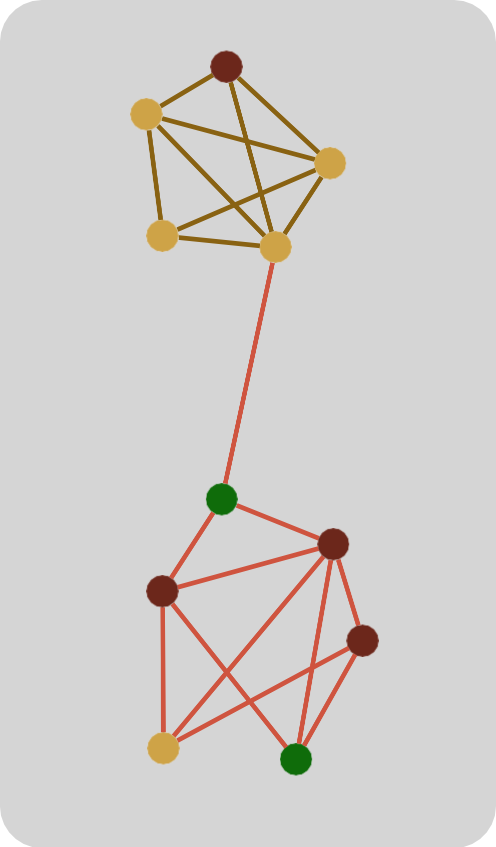
d_s = Sum of the degrees of species in module s

The term $\left(\frac{d_s}{L}\right)^2$ represents the expected fraction of interactions within module *s* for a network where interactions are randomly rewired



$$M = \sum_{s=1}^{N_m} \left[\frac{l_s}{L} - \left(\frac{d_s}{L} \right)^2 \right]$$

Measuring modularity: a practical example

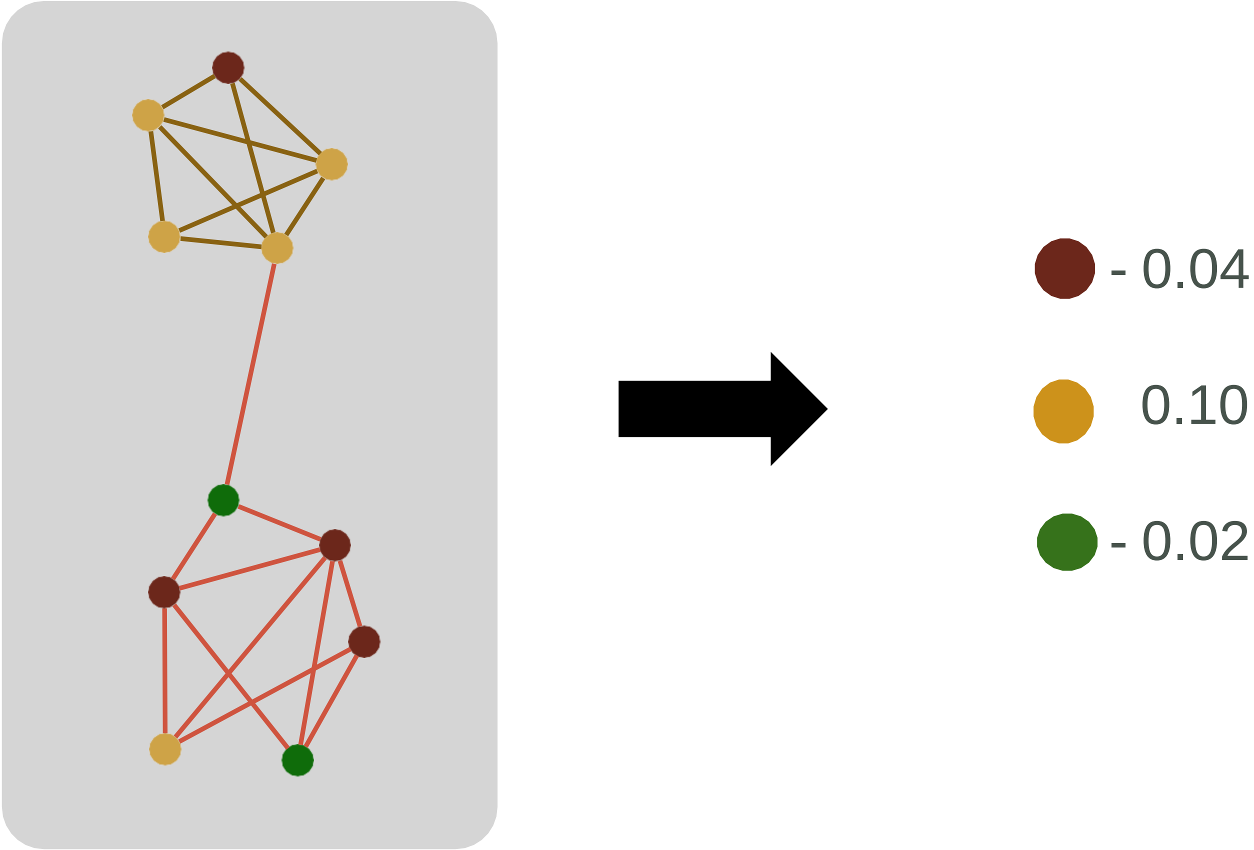


$(2/20) - (15/40)^2$

$(6/20) - (18/40)^2$

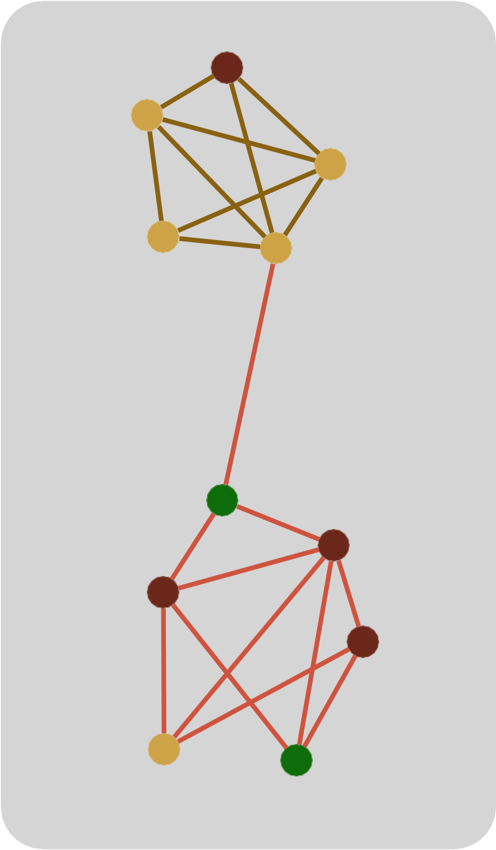
$(0/20) - (6/40)^2$

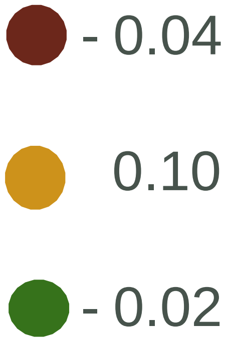
Measuring modularity: a practical example



$$M = \sum_{s=1}^{N_m} \left[\frac{l_s}{L} - \left(\frac{d_s}{L} \right)^2 \right]$$

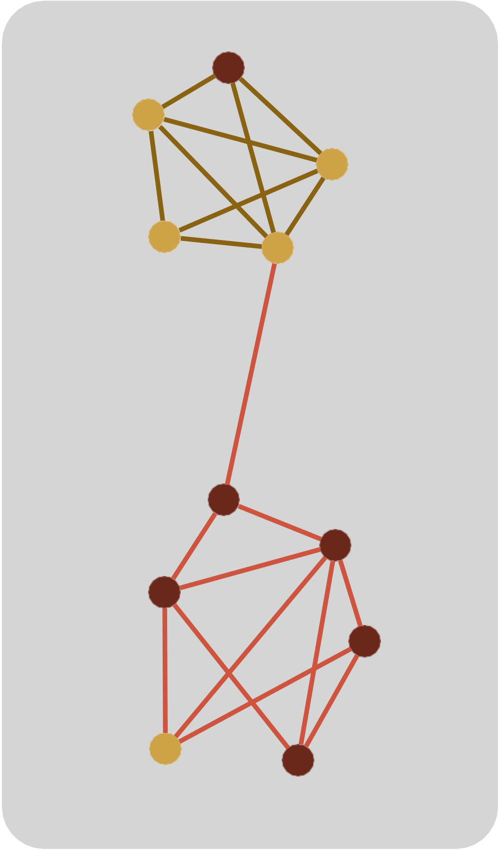
Measuring modularity: a practical example





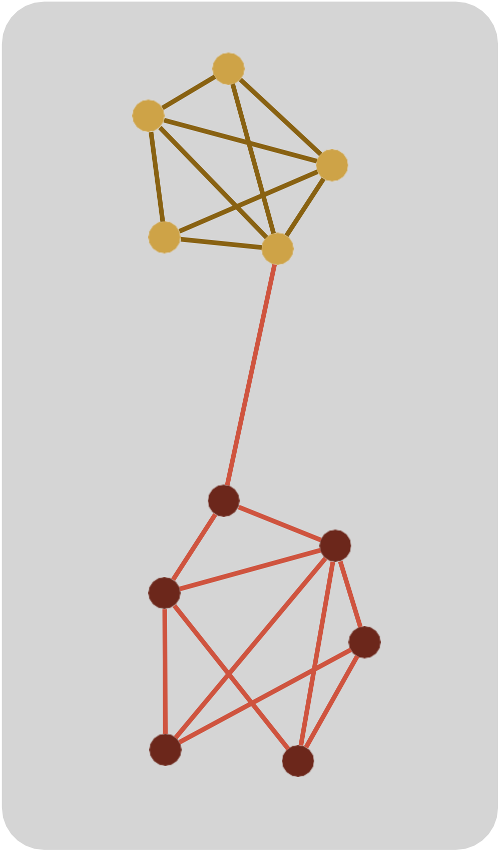
$$M = 0.04$$

Measuring modularity: a practical example



$$M = 0.16$$

Measuring modularity: a practical example

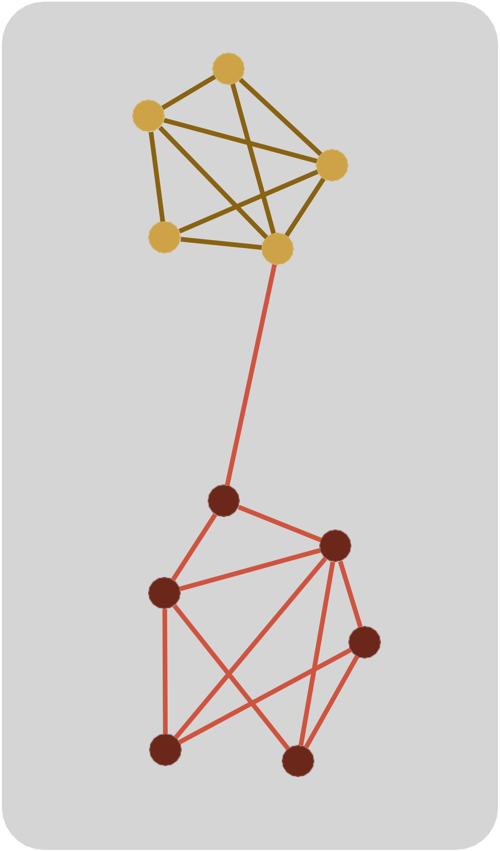


0.22

0.25

$$M = 0.47$$

Measuring modularity: a practical example

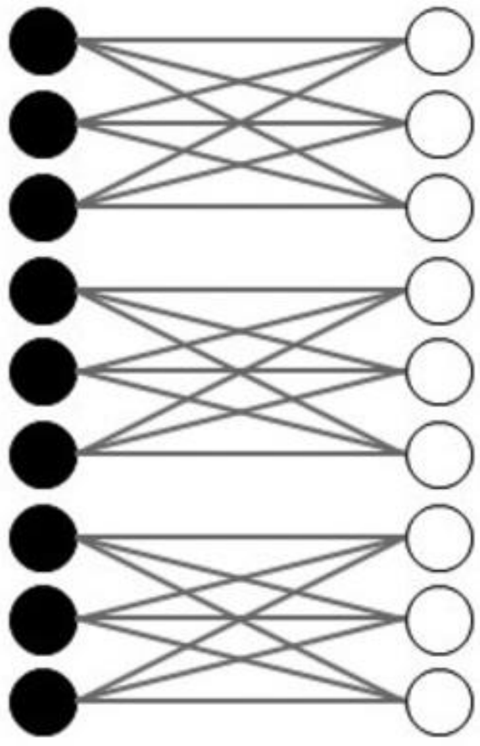


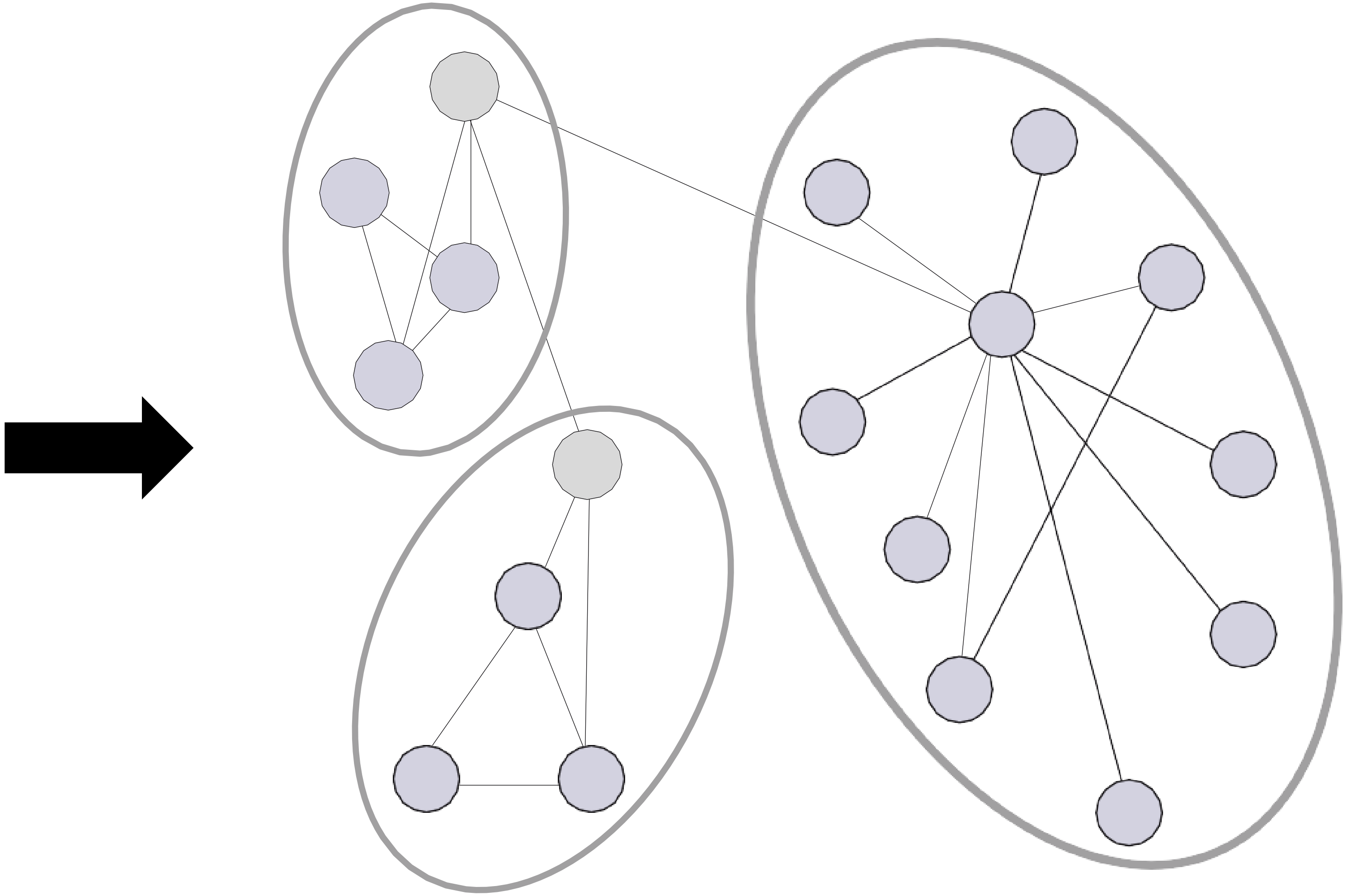
0.22

0.25

$$M = 0.47$$

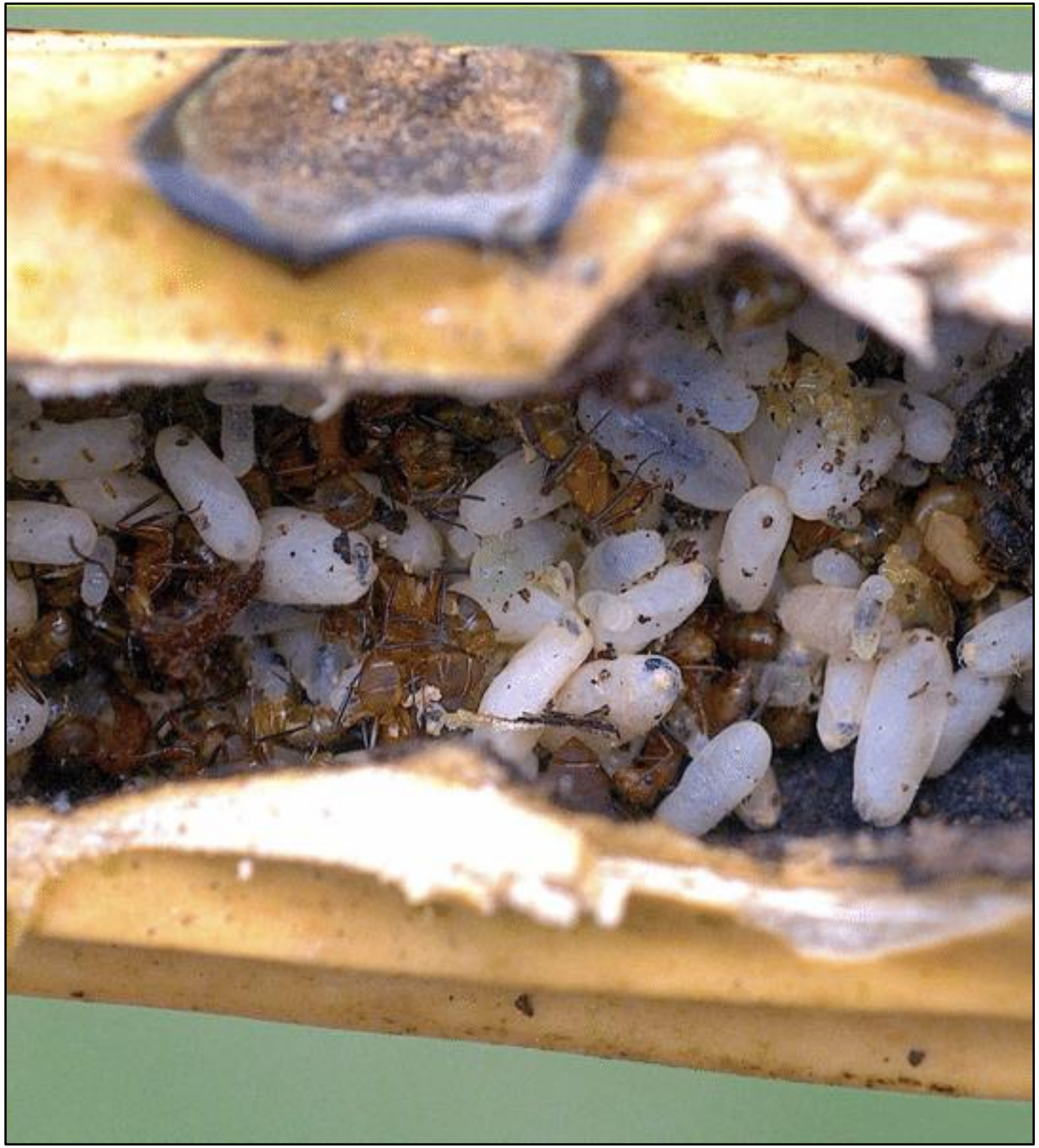
Measuring modularity: optimization algorithms





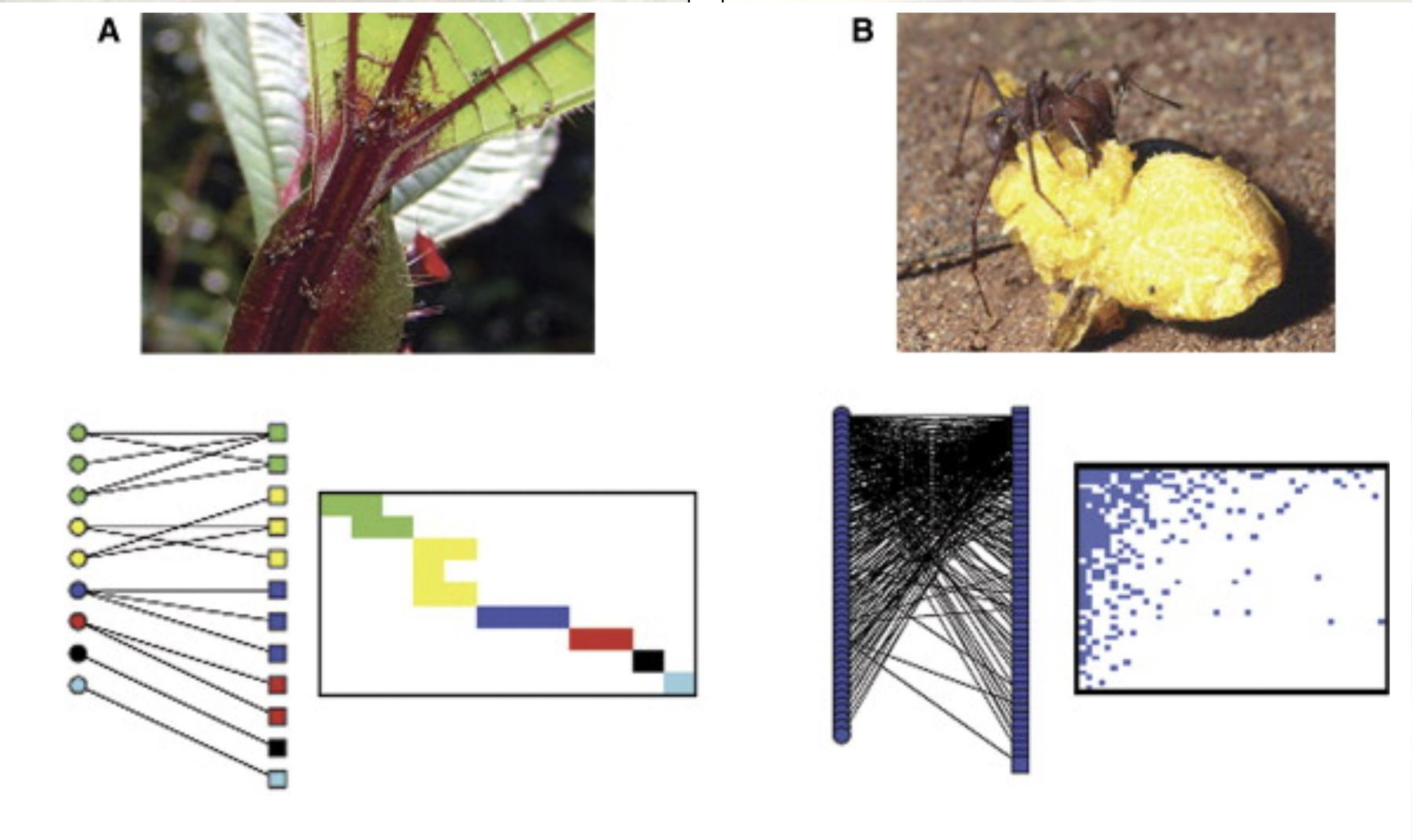
Now that we now how to quantify a modular pattern:
Where can we find it in nature?


00129405 © Mark Moffett / Minden / naturepl.com



MYRMECOPHYTES & ANTS	*Cecropia purpuracens*	*Cecropia concolor*	*Cecropia distachya*	*Cecropia ficifolia*	*Pouruma heterophylla*	*Hirtella myrmecophila*	*Hirtella physophora*	*Duroia saccifera*	*Cordia nodosa*	*Cordia aff. nodosa*	*Tococa bullifera*	*Maieta guianensis*	*Maieta poeppigi*	*Tachigali polyphylla*	*Tachigali myrmecophila.*	*Amaioua aff. guianensis*
Camponotus balzanii	11															
Azteca alfari	1															
Azteca isthmica	1	1	1	1												
Azteca aff. isthmica	1			2												
Allomerus D					23											
Allomerus prancei							5									
Allomerus aff. octoarticulata						3	70	27								
Solenops A							3	1								
Allomerus auripunctata								2		2						
Crematogaster B								1	1	1						
Azteca HC										3						
Azteca G										24	11	2				
Crematogaster D										3	2					
Azteca CO										1						
Pheidole minutula											1	93	28			
Crematogaster A							1				7	7	1			
Azteca TO											1					
Crematogaster C											3		3			
Azteca schummani														2	1	
Pseudomyrmex nigrescens														7	16	
Pseudomyrmex concolor														16	18	
Azteca D															1	
Azteca polymorpha															2	
Crematogaster E										1					1	
Azteca Q																3
Unoccupied plants	14	0	0	0	0	0	3	8	0	31	0	5	5	6	5	0

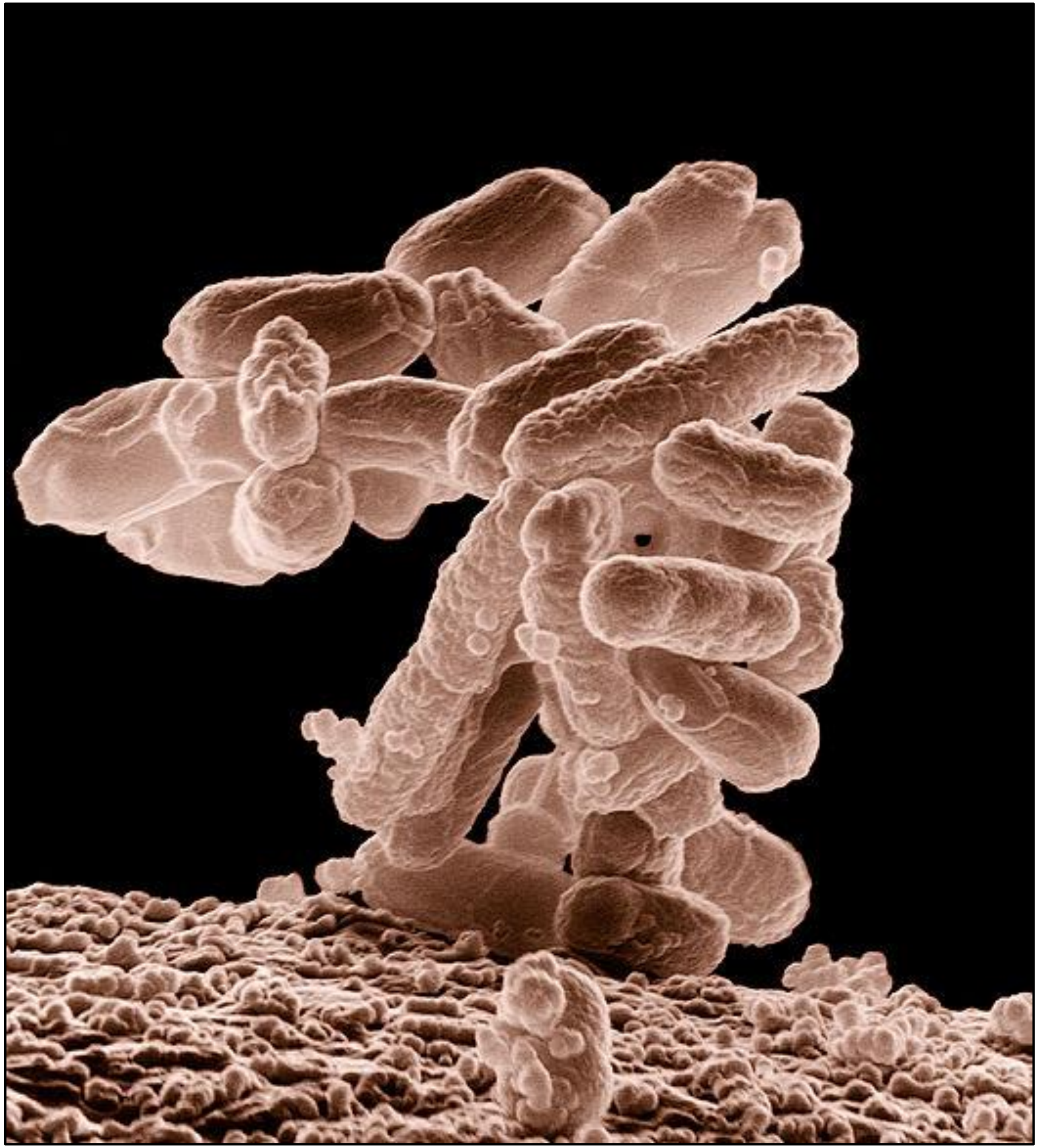
(Fonseca & Ganade *1996*)

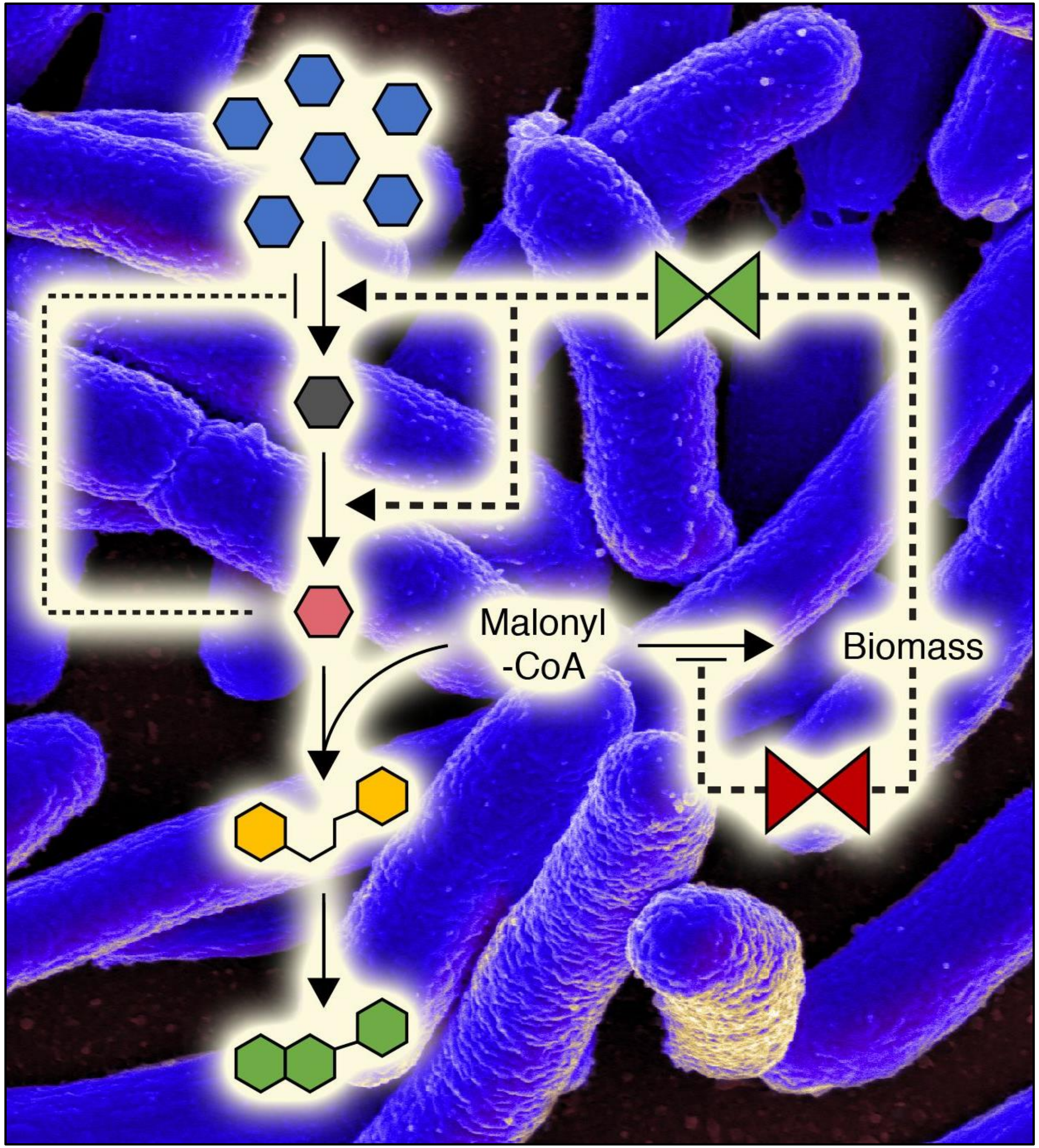


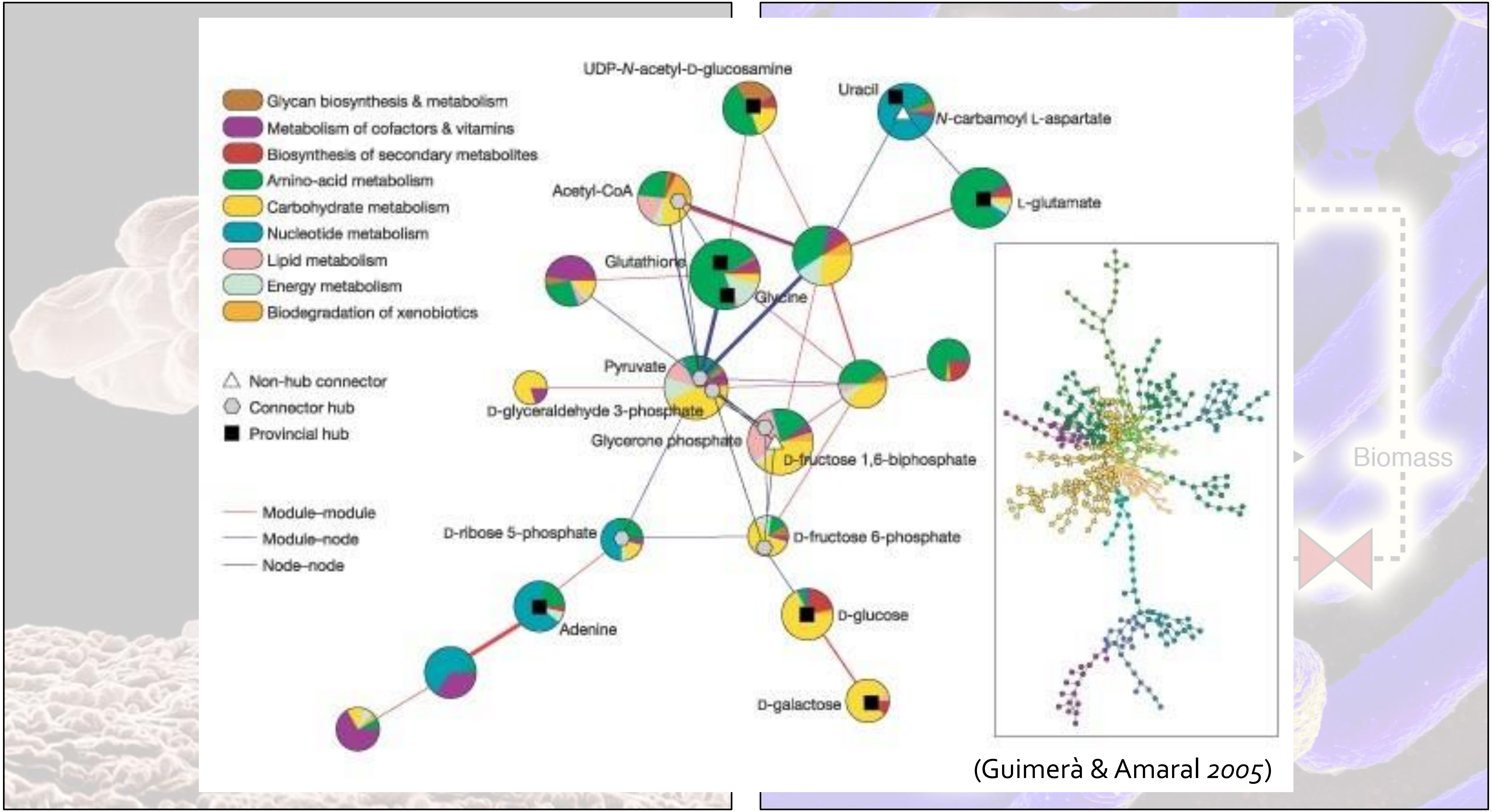
(Guimarães *et al.* 2007)


			1		2			3					4											5							6							
			Tomoplagia argentiniensis	*Tomoplagia pseudopenicillata*	*Tomoplagia fiebrigi*	*Tomoplagia reimoseri*	*Tomoplagia minuta*	*Tomoplagia grandis*	*Tomoplagia achromoptera*	*Tomoplagia brasiliensis*	*Dictyotrypeta atacta*	*Tomoplagia cipoensis*	*Tomoplagia tripunctata*	*Tomoplagia incompleta*	*Xanthaciura biocellata*	*Dictyotrypeta* sp.1	*Dictyotrypeta* sp.2	*Tomoplagia* sp.1	*Tomoplagia dimorphica*	*Tomoplagia variabilis*	*Tetreuaresta* sp.1	*Tomoplagia formosa*	*Tomoplagia punctata*	*Tetreuaresta* sp.2	*Dictyotrypeta* sp.3	*Tomoplagia bicolor*	*Tetreuaresta* sp.3	*Trupanea* sp.1	*Acrotaeniini* n.gen.1 sp.1	*Tomoplagia interrupta*	*Tomoplagia aczeli*	*Xanthaciura chrysura*	*Tomoplagia voluta*	*Acrotaeniini* n.gen.2 sp.1	*Tomoplagia rupestris*	*Tomoplagia* sp.2	*Tetreuaresta* sp.4	Total
1	V	*Cyrtocymura scorpioides*	X	X																																		2
	V	*Vernonanthura* aff. *lucida*			X	X																																2
	V	*Vernonanthura* aff. *laxa*				X																																1
	V	*Vernonanthura glanduloso-dentata*				X																																1
	V	*Vernonanthura membranacea*				X																																1
2	V	*Vernonanthura mucronulata*				X																																1
	V	*Vernonanthura phosphorica*				X																																1
	V	*Vernonanthura subverticillata*				X	X																															2
	V	*Vernonanthura westiniana*					X																															1
	V	*Vernonanthura mariana*				X	X								X																							3
	V	*Lessingianthus buddleiifolius*						X			X																											2
	L	*Proteopsis* sp.01							X		X																											2
3	V	*Lessingianthus coriaceus*							X	X	X			X																								4
	V	*Lessingianthus roseus*								X	X	X		X																								4
	V	*Lessingianthus linearis*										X																										1
	V	*Lessingianthus vepretorum*								X	X	X	X	X		X																						6
	V	*Lessingianthus rosmarinifolius*											X																									1
	V	*Lessingianthus linearifolius*											X	X																								2
	V	*Lessingianthus carduoides*											X	X																								2
	V	*Lessingianthus brevipetiolatus*											X	X																								2
	V	*Lessingianthus pumillus*												X																								1
	V	*Lessingianthus glabratus*												X																								1
	V	*Lessingianthus simplex*												X																								1
	V	*Lessingianthus* aff. *simplex*												X																								1
	L	*Eremanthus* sp.03												X																								1
	H	*Chresta* sp.01												X																								1
	V	*Lessingianthus tomentellus*											X			X																						2
	H	*Chresta sphaerocephala*												X			X	X								X												4
	V	*Lessingianthus* sp.01														X																						1
	L	*Eremanthus erythropappus*														X																						1
	V	*Lessingianthus psilophyllus*											X	X	X			X																				4
4	V	*Lessingianthus warmingianus*												X		X		X																				3
	V	*Lepidaploa spixiana*												X		X		X																				3
	V	*Echinocoryne schwenkiaefolia*												X		X		X																				3
	V	*Lessingianthus rubricaulis*												X				X																				2
	V	*Lepidaploa lilacina*												X				X																				2
	V	*Lepidaploa* aff. *rufogrisea*												X				X																				2
	V	*Lessingianthus virgulatus*													X			X																				2
	V	*Echinocoryne holosericea*														X		X																				2
	V	*Stilpnopappus bicolor*															X																					1
	V	*Lessingianthus hoveaefolius*												X	X				X	X																		4
	V	*Lepidaploa rufogrisea*												X				X			X																	3
	V	*Chrysolaena herbacea*												X				X				X																3
	C	*Centratherum punctatum*														X							X															2
	V	*Vernonanthura ferruginea*																X																				1
	V	*Lepidaploa* sp.01																X																				1
	V	*Lepidaploa salzmannii*																X																				1
	V	*Lepidaploa aurea*																			X																	1

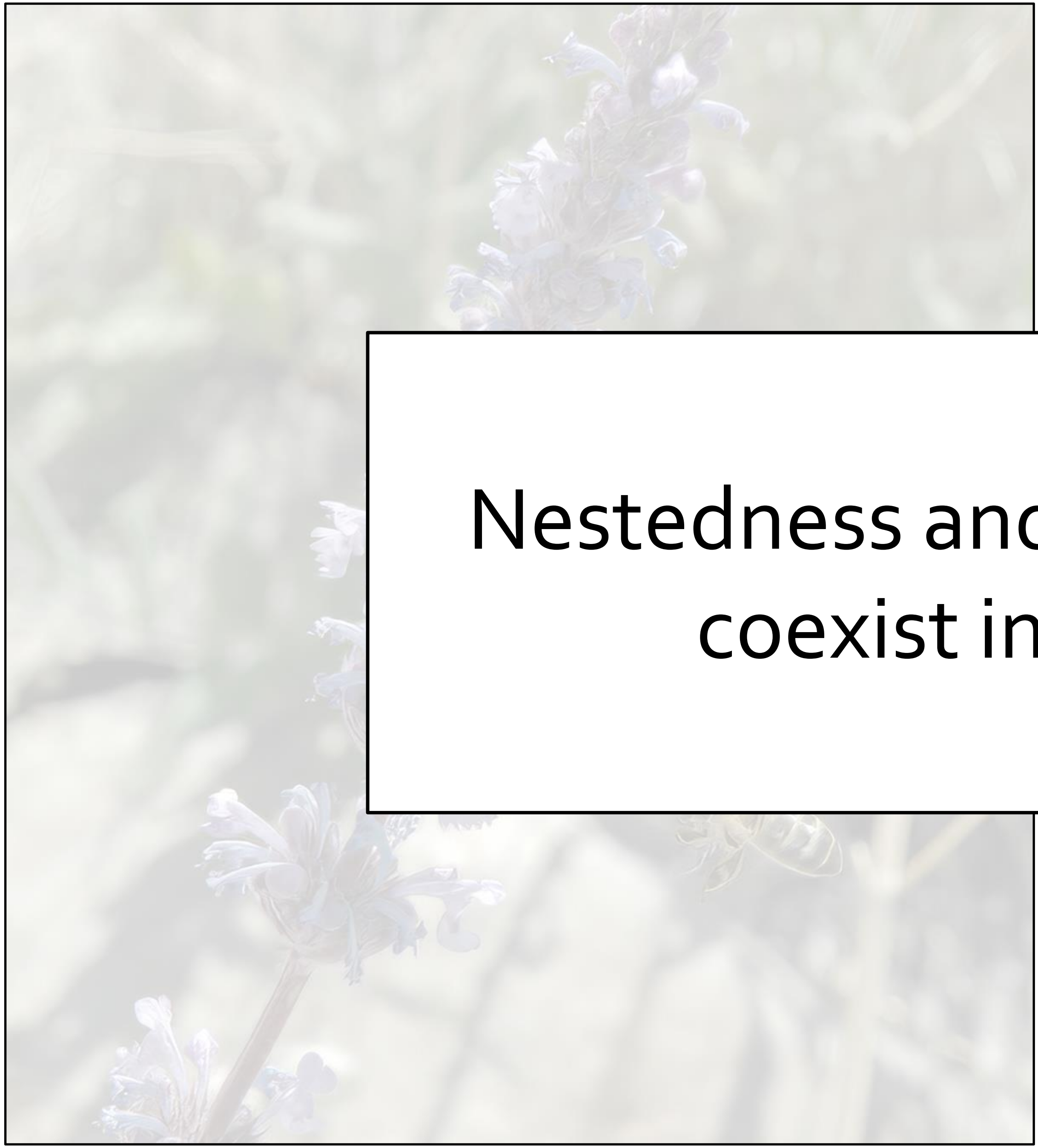
(Prado & Lewinsohn *2004*)

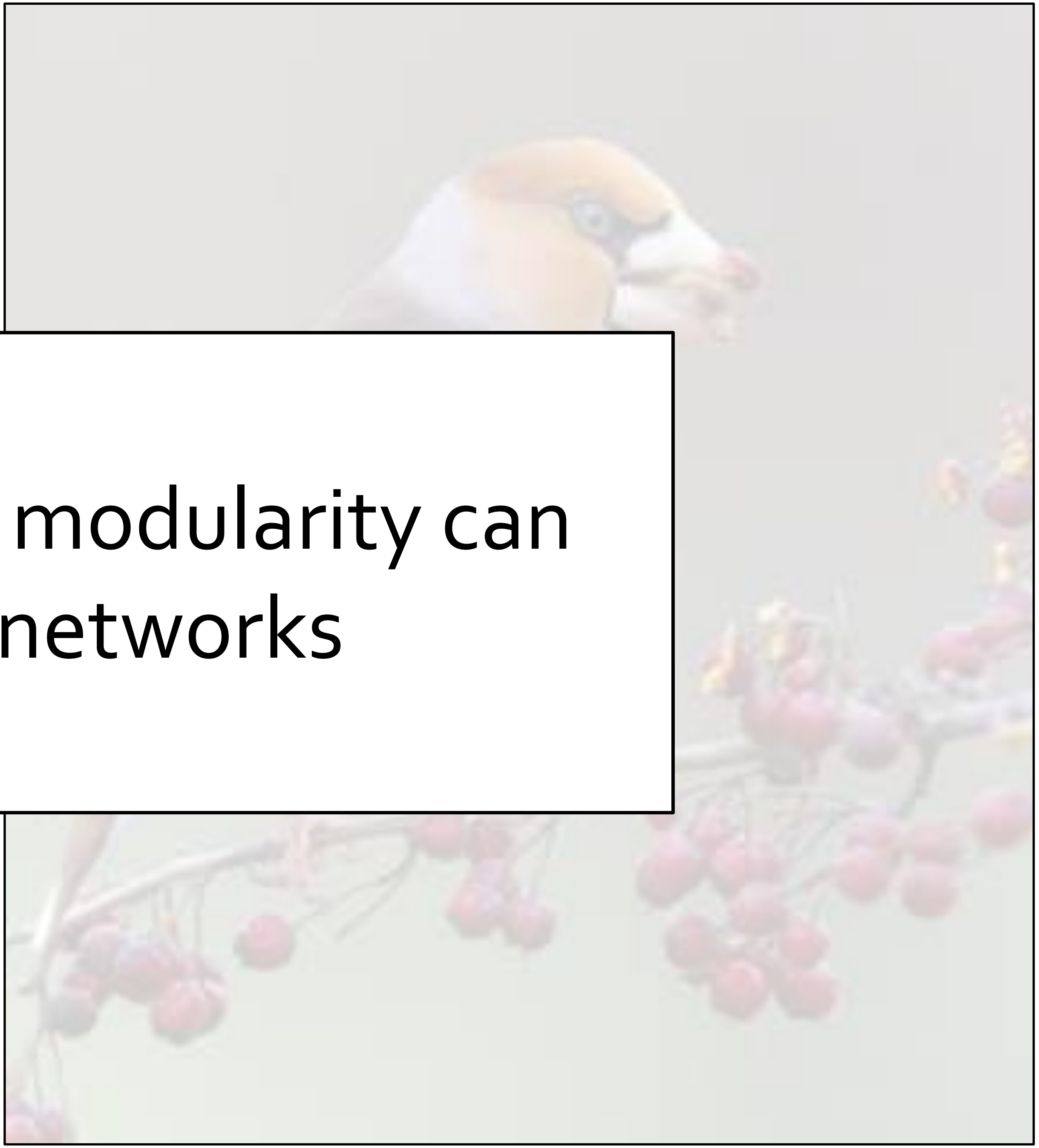



Malonyl
-CoA
Biomass

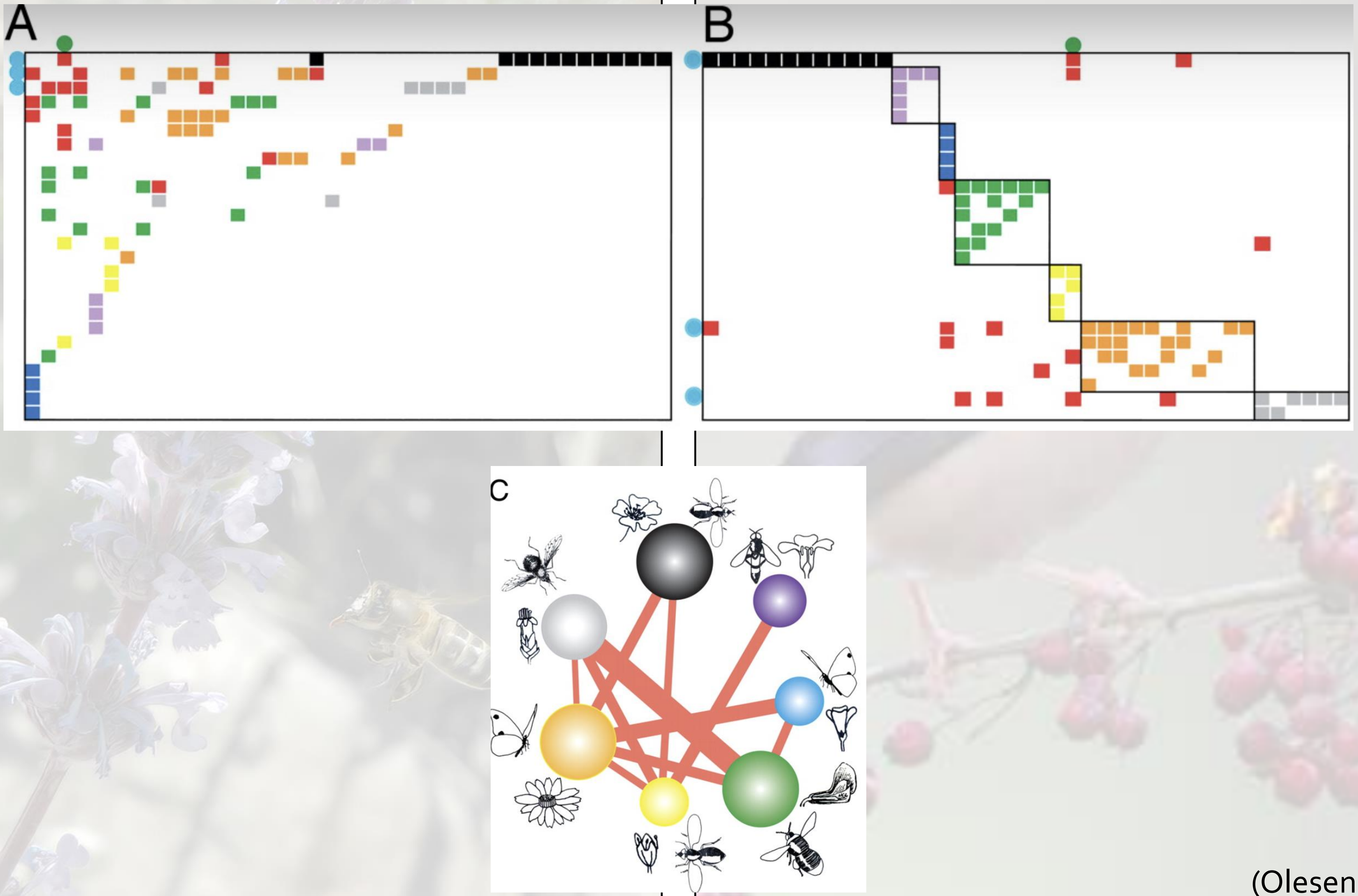


(Guimerà & Amaral *2005*)

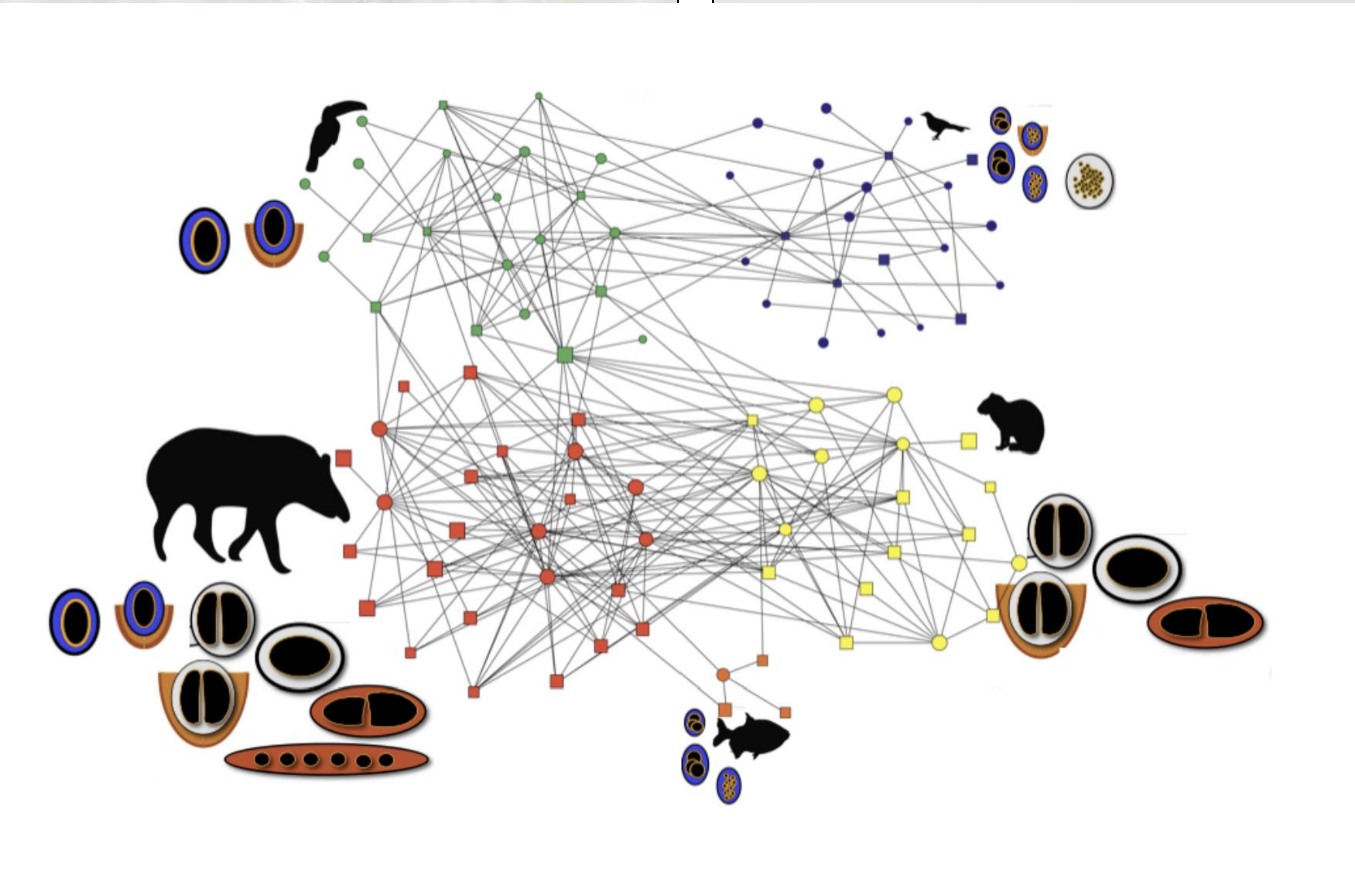





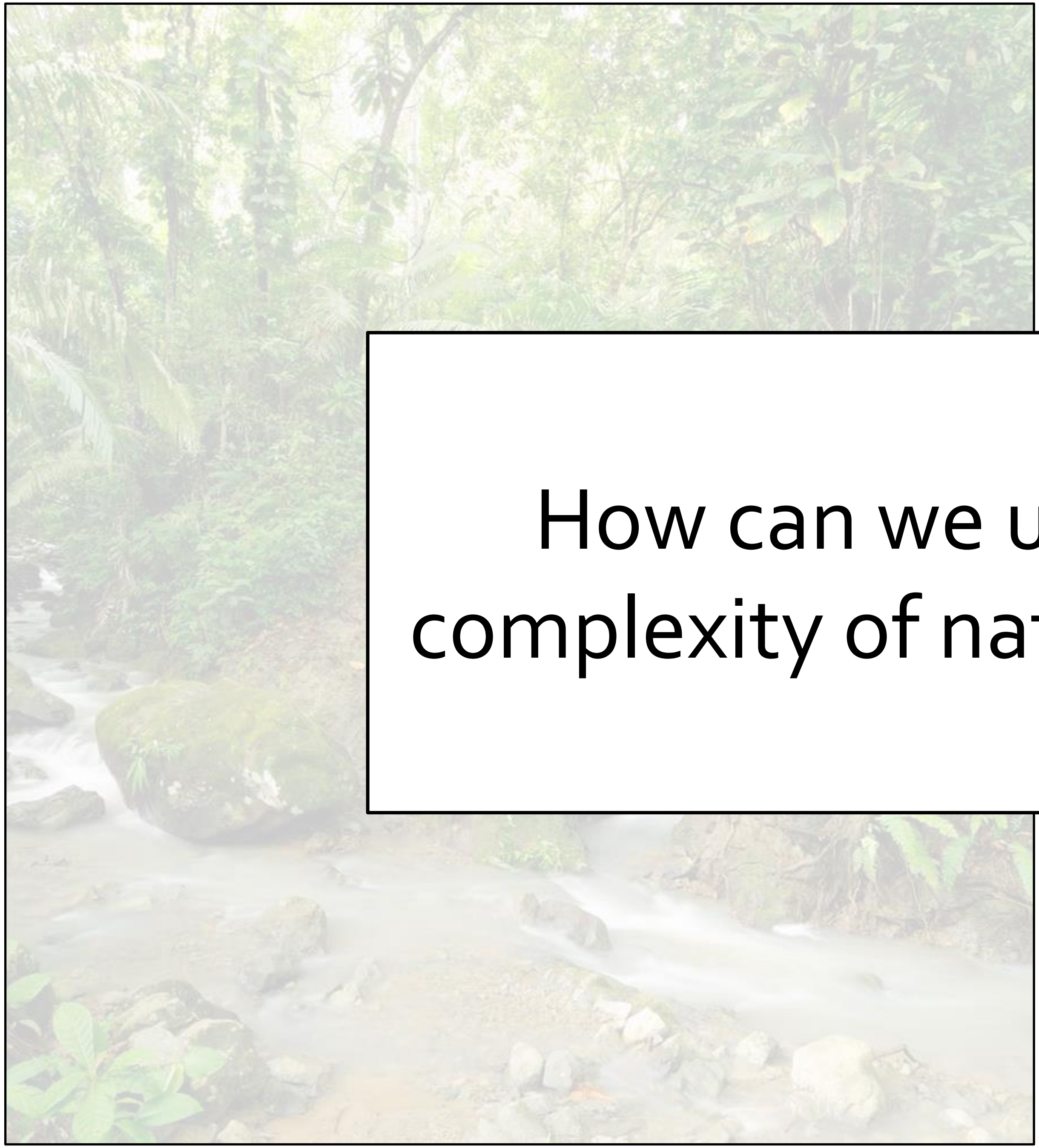
Nestedness and modularity can coexist in networks



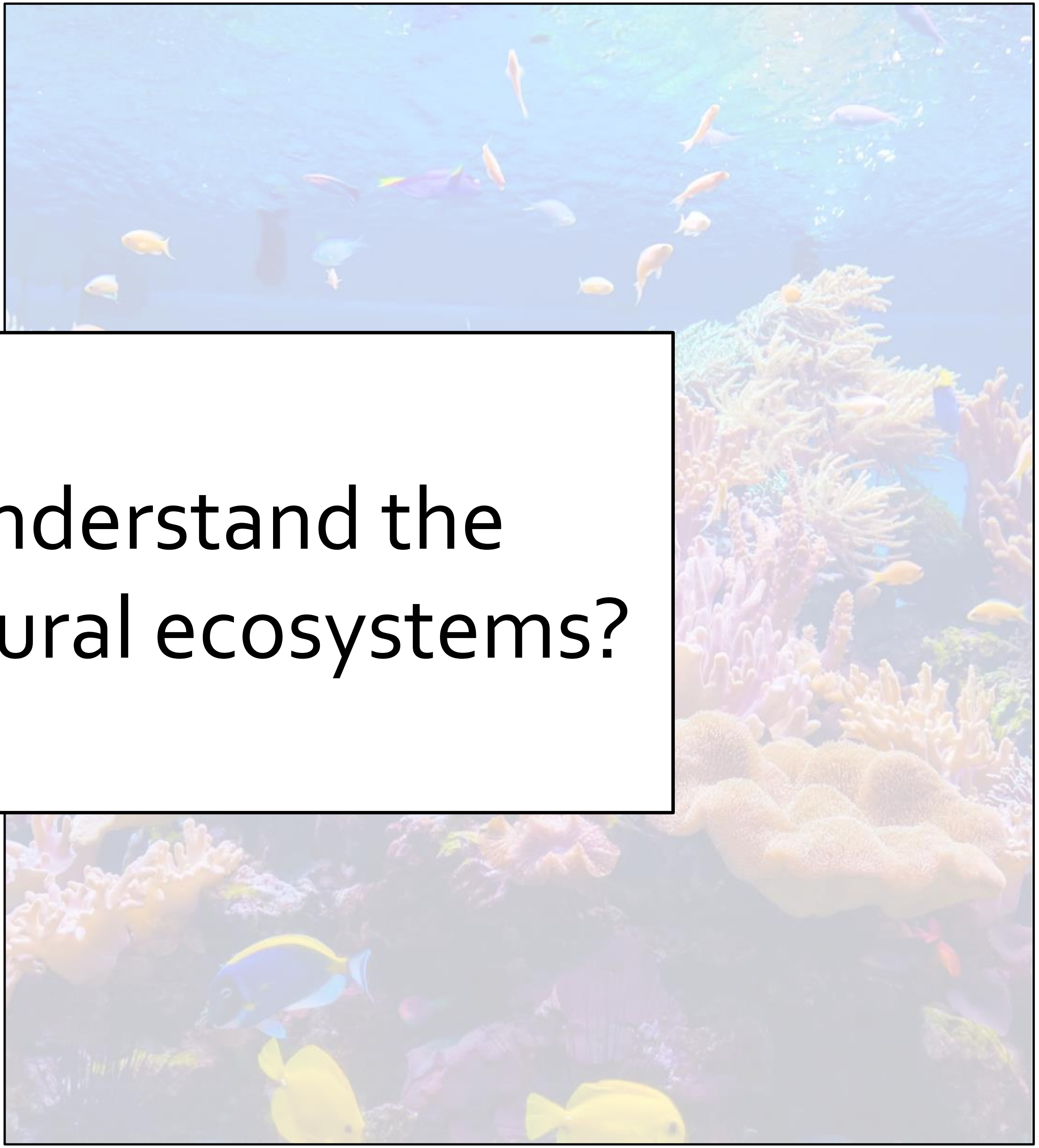
(Olesen et al. *2007*)

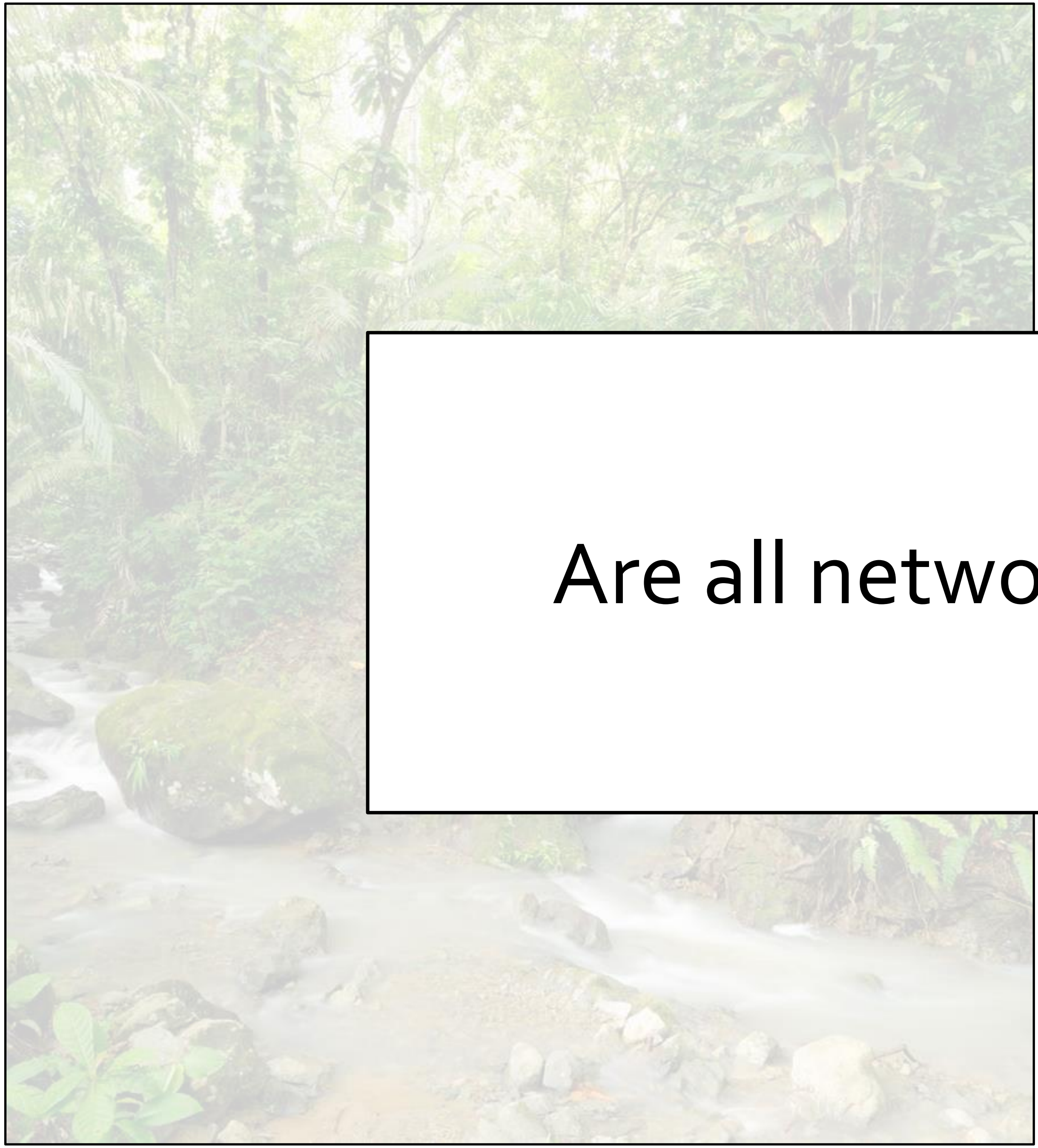


(Donatti et al. *2012*)

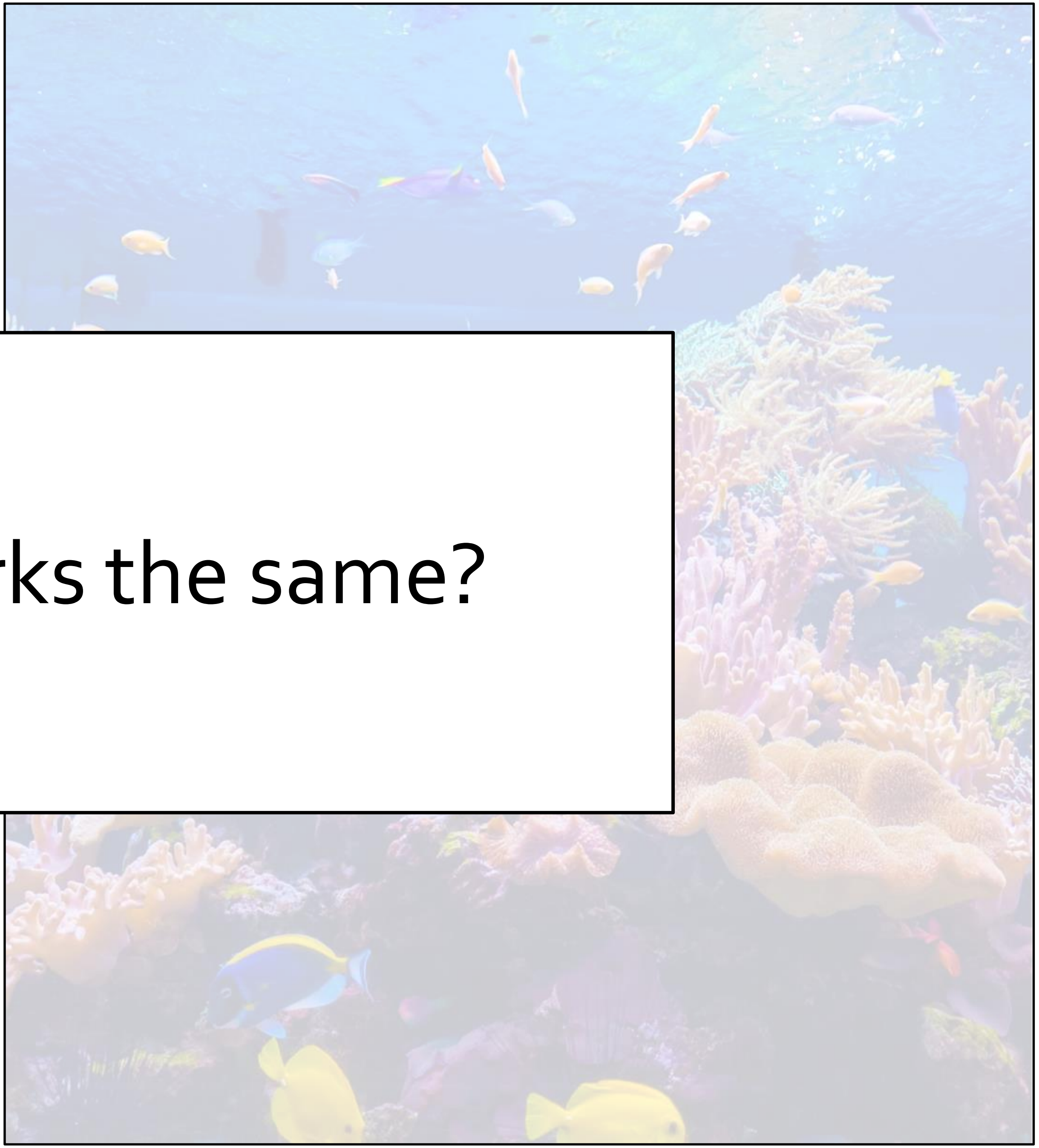


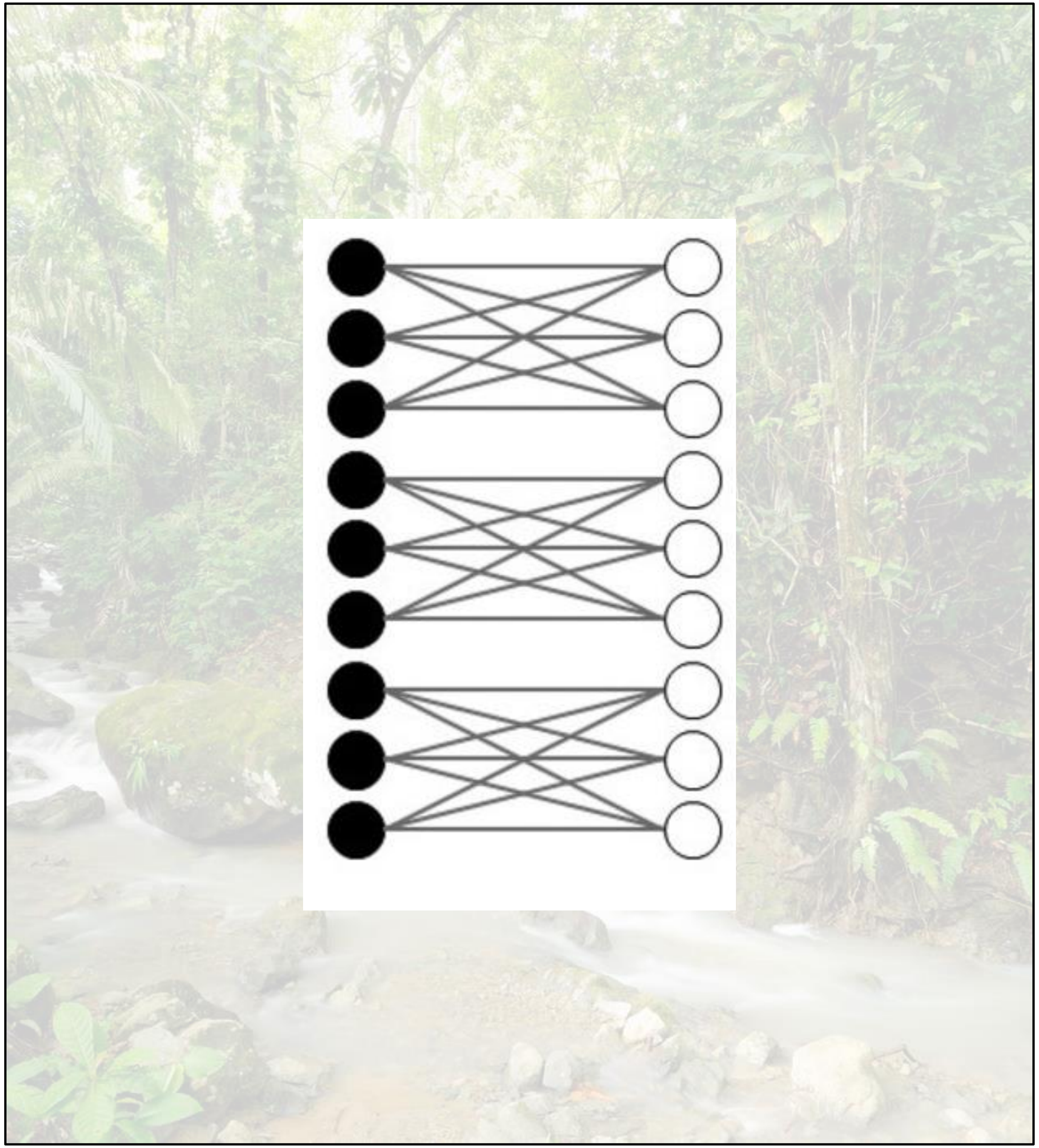
How can we understand the complexity of natural ecosystems?

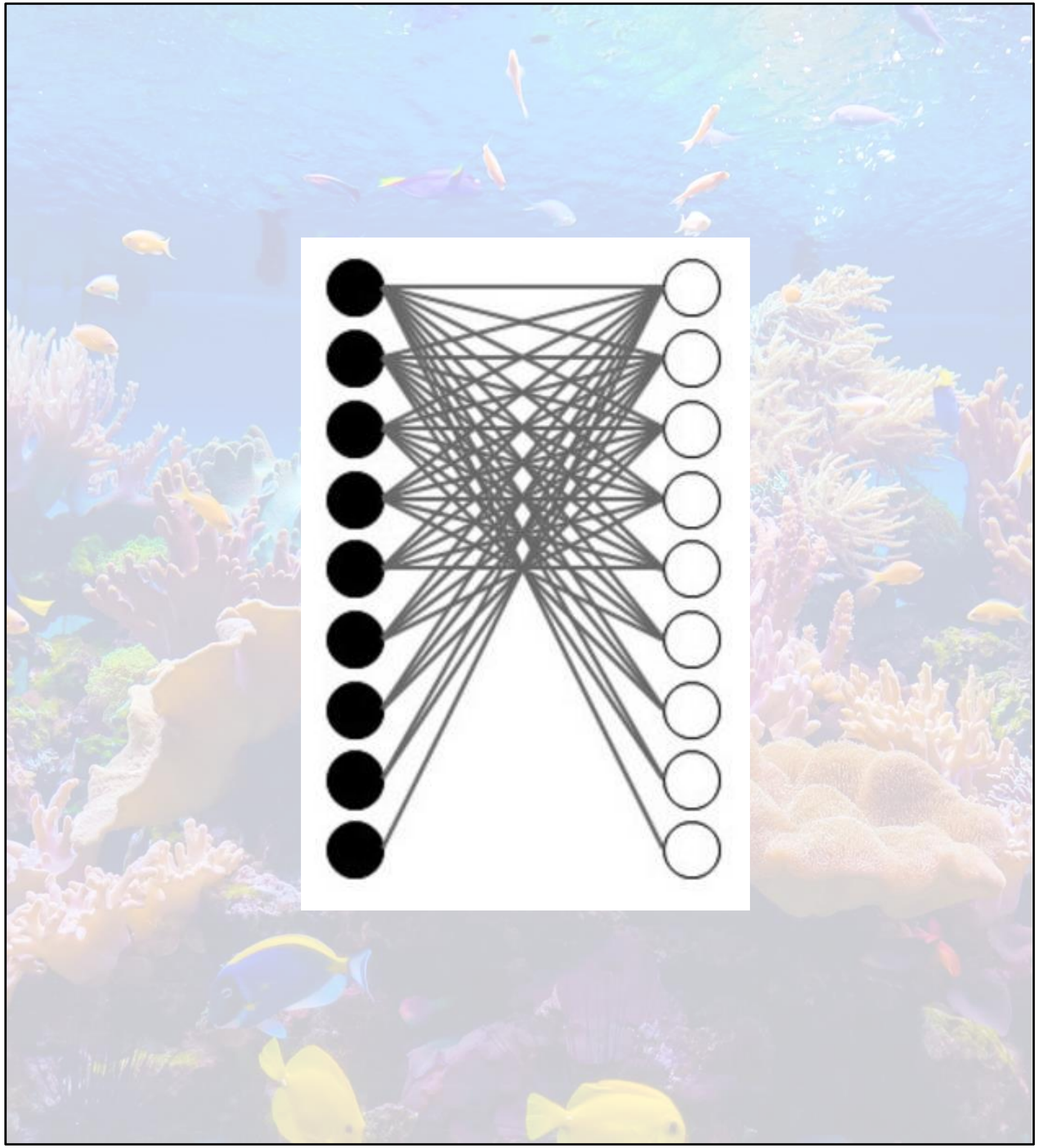




Are all networks the same?



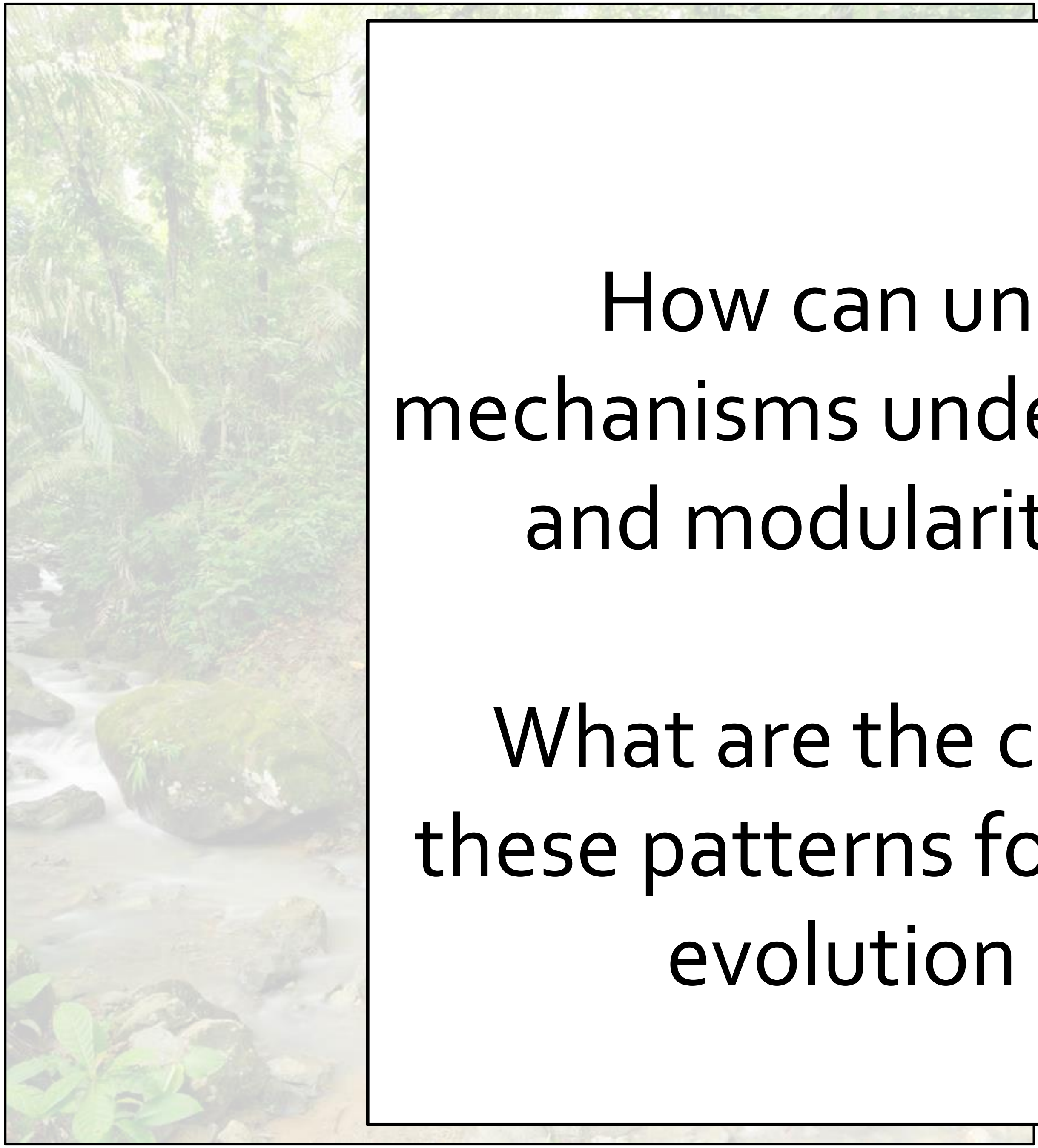




Quantifying topological patterns:

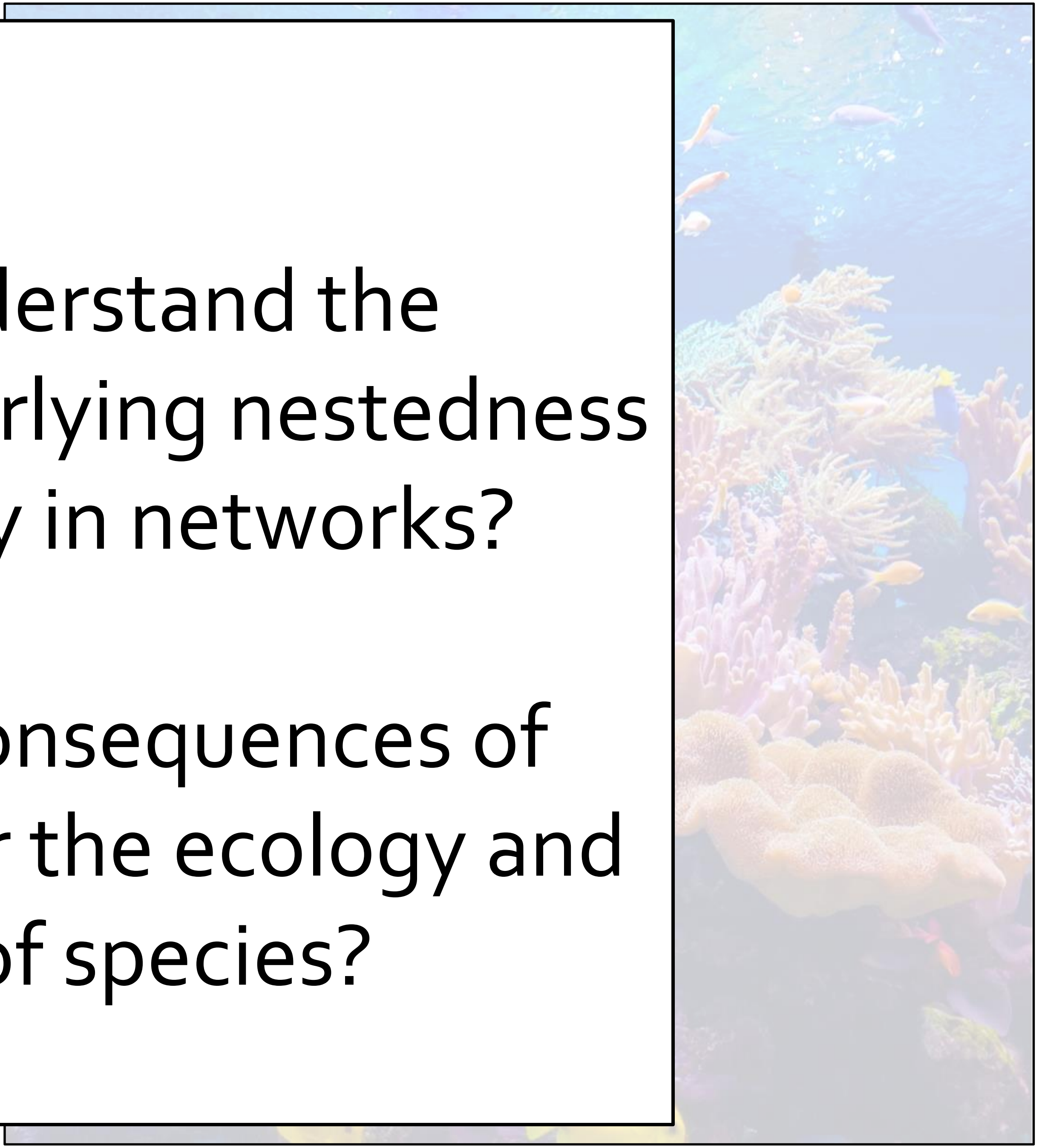
Nestedness:
Low interaction intimacy, hierarchical organization of interactions

Modularity
High interaction intimacy, tightly connected, functional groups/units



How can understand the mechanisms underlying nestedness and modularity in networks?

What are the consequences of these patterns for the ecology and evolution of species?



Topological patterns in ecological networks

BIO365 – Ecological Networks

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